



BRNO UNIVERSITY OF TECHNOLOGY

VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ

FACULTY OF MECHANICAL ENGINEERING

FAKULTA STROJNÍHO INŽENÝRSTVÍ

INSTITUTE OF MACHINE AND INDUSTRIAL DESIGN

ÚSTAV KONSTRUOVÁNÍ

DEVELOPMENT OF FILM THICKNESS IN ELASTOHYDRODYNAMICALLY LUBRICATED COMPLIANT CONTACTS

VÝVOJ TLOUŠTKY FILMU V ELASTOHYDRODYNAMICKY MAZANÝCH PODDAJNÝCH KONTAKTECH

DOCTORAL THESIS

DIZERTAČNÍ PRÁCE

AUTHOR

AUTOR PRÁCE

Ing. Jiří Křupka

SUPERVISOR

ŠKOLITEL

prof. Ing. Ivan Křupka, Ph.D.

BRNO 2023

ABSTRACT

Lubrication of polymer machine elements, such as polymer gears, is a current subject of the interest in multiple mechanical engineering applications in terms of simulation of lubrication processes and optimization of tribological conditions to increase the performance of these elements. The present dissertation thesis deals with an experimental study of the development of the fluid-film thickness in elastohydrodynamically lubricated compliant contacts. The main aim of the thesis is to clarify the formation of film thickness in compliant contacts operating in the transition region (TR) between isoviscous-elastic (I-E) and piezoviscous-elastic (P-E) modes of the elastohydrodynamic lubrication (EHL) regime. For this purpose, laboratory experiments were performed on the rotary optical tribometer in ball-on-disc configuration, and the method of optical chromatic interferometry was implemented. Both the rheological response of the lubricant and the constitutive viscoelastic response of the polymer to the formation of the fluid-film thickness were experimentally analyzed. Based on the results, the low validity of the I-E and P-E film thickness prediction models in the TR region was documented. The findings presented in this thesis extend the knowledge of the formation of lubricant film in compliant contacts operating in the EHL regime and can thus be considered in the design of polymer machine elements or in lubrication management.

KEYWORDS

Polymer gears, compliant contact, elastohydrodynamic lubrication, fluid-film thickness, transition region, optical chromatic interferometry

ABSTRAKT

Mazání polymerních strojních součástí, jako jsou polymerní ozubená kola, je v současné době předmětem zájmu mnoha strojírenských aplikací z hlediska simulace mazacích procesů a optimalizace tribologických podmínek pro zvýšení výkonnosti těchto součástí. Předkládaná disertační práce se zabývá experimentální studií vývoje tloušťky kapalinového filmu v elastohydrodynamicky mazaných poddajných kontaktech. Hlavním cílem práce je objasnit formování tloušťky filmu v poddajných kontaktech v přechodové oblasti (TR) mezi isoviscous-elastic (I-E) a piezoviscous-elastic (P-E) režimem elastohydrodynamického mazání (EHL). Za tímto účelem byly provedeny laboratorní experimenty na rotačním optickém tribometru v konfiguraci kulička na disku, ve kterých byla implementována metoda optické chromatické interferometrie. Experimentálně byl analyzován vliv jak reologické odezvy maziva, tak konstitutivní viskoelastické odezvy polymeru na tvorbu tloušťky kapalinového filmu. Na základě výsledků byla dokumentována nízká platnost predikčních modelů tloušťky filmu I-E a P-E v oblasti TR. Výsledky prezentované v této práci rozšiřují poznatky o tvorbě mazacího filmu v poddajných kontaktech pracujících v režimu EHL a mohou tak být zohledněny při návrhu polymerních strojních součástí nebo při řízení mazacích procesů.

KLÍČOVÁ SLOVA

Polymerní ozubení, poddajný kontakt, elastohydrodynamické mazání, tloušťka filmu, přechodová oblast, optická chromatická interferometrie

BIBLIOGRAPHICAL REFERENCE

KŘUPKA, Jiří. *Development of film thickness in elastohydrodynamically lubricated compliant contacts*. Brno, 2023. 143p., PhD thesis. Brno University of Technology, Faculty of Mechanical Engineering, Institute of Machine and Industrial Design. Supervisor: prof. Ing. Ivan Křupka, Ph.D.

ACKNOWLEDGEMENT

I would like to express my thanks to my supervisor prof. Ing. Ivan Křupka, Ph.D. for valuable advice, recommendations, and discussions over the topic of my dissertation thesis as well as for his willingness, tolerance and assistance during my study at the Institute of Machine and Industrial Design, Brno University of Technology. Special thanks should go to Ing. Kryštof Dočkal for excellent cooperation, his helpful advice and discussions over our scientific papers and preparation of laboratory measurements. Also, I wish to express my thank to my colleagues Ing. Daniel Kvarda Ph.D., Ing. Josef Frýza Ph.D., Ing. Vojtěch Polnický, and Zdeněk Růžička for their help and good atmosphere during my studies. I also owe thanks to my family for their long-term support and understanding throughout my studies.

STATEMENT

I hereby declare that I have written the PhD thesis *Development of film thickness in elastohydrodynamically lubricated compliant contacts* on my own according to advice of my supervisor prof. Ing. Ivan Křupka, Ph.D., and using the sources listed in references.

In Brno, 25/09/2023

.....
Ing. Jiří Křupka

CONTENTS

1	INTRODUCTION	11
2	STATE OF THE ART	12
2.1	Polymer gears, engineering polymers, and viscoelasticity	12
2.2	Lubrication regimes, EHL modes, and lubricant rheology	17
2.3	Formation of film thickness in soft EHL contacts – numerical approach	22
2.4	Formation of film thickness in soft EHL contacts – experimental approach	32
3	ANALYSIS AND CONCLUSION OF LITERATURE REVIEW	41
3.1	Analysis of literature review	41
3.2	Main conclusions of literature review	45
4	RESEARCH AIM OF THESIS	48
4.1	Scientific questions	48
4.2	Hypotheses	49
4.3	Thesis layout	50
5	MATERIALS AND METHODS	52
5.1	Selection of specimens	54
5.2	Selection of lubricants	57
5.3	Preparation of tribological experiments	58
5.4	Experimental apparatus	62
5.5	Optical interferometry method	63
5.6	Overview of experimental conditions	64
6	RESULTS AND DISCUSSION	66
7	CONCLUSIONS	124
	LIST OF PUBLICATIONS	127
	Papers published in journals with impact factor:	127
	Papers in conference proceedings:	127
	REFERENCES	128

NOMENCLATURE	139
LIST OF FIGURES	141
LIST OF TABLES	143

1 INTRODUCTION

Over the decades, metals have been the preferred materials for engineering applications, not only due to their stiffness, strength, and availability, but also with respect to the manufacturing process capabilities associated with the achievable accuracy of the machine elements. Plastics gradually began to replace traditional materials such as steels, non-ferrous metals, glass, paper or wood after WWII when the first phase of the expansion of thermoplastics and thermosets into many areas of industry on a global scale occurred. One of the main efforts was to expand the use of thermoplastics – polymers in engineering applications, especially in the military and automotive, where metals dominated at the time. Over the past thirty years, intensive research of polymer materials has led to the development of new materials and the improvement of the material properties of engineering polymers known. This has greatly accelerated the process of replacing steel machine elements, such as bearings, cams and gears, with polymers. The use of polymers in a wide range of engineering products has demonstrated their importance, their benefits to society and their widespread use in the products of our daily life. However, the question is whether they can withstand demanding conditions thanks to lubrication. Lubrication of the machine elements, such as gears, is one of the fundamental problems in the field of the mechanical engineering. These include transmissions, differentials and mechanisms which operate under demanding operating and environmental conditions (e.g., in the automotive and aerospace industries), where dynamics, contact mechanics and tribology interact. Polymer lubrication is the subject of current interest in many engineering applications where compliant materials are engaged. One of the main efforts is to integrate the polymer gears into high-performance applications, where dry conditions are unacceptable, and lubrication is desirable due to high loads, speeds or operating temperatures. Under these conditions, it is necessary to ensure a full separation of the rubbing surfaces, as in the elastohydrodynamic lubrication regime, by coherent film thickness of the lubricant to reduce friction and wear and thus prolong the service life of such machine elements. However, this poses challenges in the qualitative and quantitative description of the lubrication processes that occur between the non-conformal rubbing surfaces. Therefore, a subject of the interest of the solved scientific problem is to experimentally analyze the formation of the fluid-film thickness in compliant contacts operated in the elastohydrodynamic lubrication regime, covering the framework of the basic oriented research. From an economic point of view, the savings in financial costs and machining time for the production of polymer gears compared to their steel equivalents is a major reason for the interest in this topic. From an engineering point of view, the topic is relevant in terms of increase in performance of such machine elements and the expansion of the composite polymer materials into the areas of the mechanical engineering industry, where metal materials have so far dominated. Finally, from a scientific point of view, the findings of the basic oriented research carried out will broaden the awareness of tribological processes in compliant contacts lubricated with natural, mineral, and synthetic oils.

2 STATE OF THE ART

2.1 Polymer gears, engineering polymers, and viscoelasticity

The history of polymer gears dates back to the mid-20th century, when the long-term process of replacing of conventional steel machine elements initiated [1], which continues to this day. Initially, polymer gears were used to transfer motion rather than to transmit torque due to their weak mechanical characteristics. Today, the less demanding applications of polymer gears include kitchen appliances, garden equipment, IT hardware, etc., where dry contact conditions or grease lubrication are commonly ensured.

Nowadays, the operational requirements for machine elements are rapidly increasing along with the objective of increasing their efficiency and reducing manufacturing costs. This is especially the case for the polymer gears, due to their low-cost production, low mass, chemical resistance to oils and acids or the ability to damp impacts and vibrations. The downside may be a certain ecological burden, but the society-wide benefits of polymer gears are indisputable.

With respect to the geometry of the gear teeth, the most commonly used configurations of polymer gears include spur, helical, bevel, and worm gearsets whose shafts can be parallel, non-parallel and non-intersecting. In practice, different materials are often used in the gearset, with the pinion and the wheel gear usually made of metal and polymer material, respectively. This represents the condition when one of the gears (contact solids or bodies) is deformed more than the other during the meshing cycle. In general, two gearsets are mainly used: polymer/polymer and steel/polymer. Current manufacturing options of polymer gears include centrifugal or gravitational casting, machining, but the most common is injection molding [2].

The production accuracy of steel gears is usually described by an accuracy grade according to ISO 6336 [3] and ANSI/AGMA 2001-D04 [4] standards where the accuracy of polymer gears corresponds to $ISO \geq 7$ and $AGMA \geq 10$, respectively. Standards for the design of polymer gears have not yet been unified worldwide (2023/08). At present, there are two main standards: German VDI 2736 [5] and Japanese JIS B 1759 [6]. Contrary to [3],[4], these standards additionally address the prediction of temperature on the tooth flank and face.

Commonly used engineering polymers for gears include polyacetals (POM), polyamides (PA), polyurethanes (PU), etc. Polymers can exist in pure or modified form. However, in order to achieve desired material properties, they are usually modified with other polymers and additives to create additional chains and cross-links. An example of this is the widespread use of polymers containing glass-fibers which improve the mechanical and

material properties of the polymer [7]. Unfortunately, the amount of glass-fibers deteriorates the tribological properties, especially the wear rate [8],[9].

Material properties of selected engineering polymers are summarized in Table 1. These often correspond to the interval of values depending on the composition (polymer modifiers, batches, etc.) declared by individual manufacturers. Polyether-ether-ketone (PEEK) can be considered as one of the most suitable materials for the high-performance applications [10]-[12]. This is mainly due to its high service temperature (T_s), low thermal expansion (β), and high elastic modulus (E), see Table 1. The E for polymers is usually given in hundreds or thousands of megapascals, while for steels, $E \approx 210$ GPa is commonly used [13]. In addition, an important property of individual polymers is the glass-transition temperature (T_g), which characterizes the constitutive behavior of the material.

Table 1. Material properties of selected polymers and 100Cr6.

Properties / polymer	PA 66	PEEK	POM	PU	PTFE	100Cr6
Maximal service temperature, T_s (°C)	80	250	85	80	250	–
Glass-transition temperature, T_g (°C)	47	143	-40	-60	-100	–
Thermal conductivity, λ (W/m °C)	0.25	0.28	0.27	0.03	0.23	46.6
Thermal expansion, β (10^{-6} °C)	88	55	93	150	145	12
Elastic modulus, E (GPa)	2.50	3.75	2.80	1.85	0.67	210
Yield strength, R_e (MPa)	90	130	80	90	15	700
Poisson's ratio, ν (-)	0.41	0.40	0.35	0.48	0.46	0.30

Polymer materials can also refer to the so-called internal solid lubricants or dry film lubricants, such as polytetrafluorethylene (PTFE) self-lubricating coating, see Table 1. This composite material reduces temperature and friction; unfortunately, it also reduces mechanical properties of gears and increases wear rate. Other self-lubricating coatings include diamond-like hard carbon (DLC), molybdenum disulfide (MoS_2), etc. [9],[10],[14].

The contact of the teeth of gears represents the contact of two non-conformal surfaces that mesh in a very small area, which usually results in high contact pressures (p , (up to several gigapascals)) [15]. This applies to steel gears which can be considered as a contact of the rigid bodies. Conversely, polymer gears have a significantly lower stiffness [9], which results in a significant deformation of the contact surfaces and the contact is compliant rather than rigid. For the steel gears, the Hookean behavior of the material is assumed, where the elastic deformation results in a change in the curvature of the involute curve of the tooth and thus in an enlargement of the contact area in interference of gear teeth [10]. However, for the polymer gears, an inelastic deformation is assumed [8].

In contrast to metals, polymers exhibit viscoelastic behavior of the material [16]. A phenomenon of viscoelastic behavior is achieved after the material exhibits a non-linear (non-Hookean) response in time in relation to the given constant stress or strain corresponding to the stress–strain dependence [16],[17]. This is associated with the rearrangement of macromolecules in the internal structure; they can occupy different conformations depending on the stress, temperature, and time. The viscoelastic response of the material is frequently numerically simulated by standard linear solid models (SLS) such as Maxwell's, Zener's and Kelvin–Voight's models, where dampers and springs in serial or parallel arrangement invoke a time-varying response [18],[19]. Thus, the original elastic modulus is substituted by the relaxation or complex modulus with regard to the temperature or time (or frequency) domain, respectively. The complex modulus (E^*) includes two main components: the elastic part, the storage modulus (E'), as well as the viscous part, the loss modulus (E''), see Eqs. (1, 2).

Complex modulus E^* :

$$E^* = E' + i E'' \quad (1)$$

Loss factor $\tan(\delta)$:

$$\tan \delta = \frac{\sin \delta}{\cos \delta} = \frac{E''}{E'} \quad (2)$$

The ratio between them represents the damping / loss factor ($\tan \delta$), corresponding to the stored energy in the material and the loss energy converted (energy dissipated due to internal friction of molecules in the polymer structure) to the heat after the polymer is tempered from glassy state ($T \ll T_g$) and viscoelastic state ($T \leq T_g$) to the rubbery plateau where $T > T_g$. In the glassy state, the E is only slightly affected by temperature, which covers the operating region of polymers where the theory of linear viscoelasticity is applicable. In the rubbery state of the polymer, large deformations can be achieved with small forces [20], which corresponds to a sharp decrease in the E . In the melt state, the polymer flows mainly as a result of breaking the bonds in the internal structure and accelerating the motion of individual molecules after the operating temperature reaches the melting temperature (T_m) [21].

The individual states define a macromolecular arrangement and the reciprocal motion of molecules of the polymer chains (bonds) relative to the internal polymer structure and operating conditions [20]. According to the internal structure, the amorphous (PMMA, PC) [22], semi-crystalline (PA66, POM, PEEK) [21],[23] polymers demonstrate a significantly different behavior in individual states with the rising temperature where the amorphous polymers prove a substantial temperature reliance rather than semi-crystalline and crystalline ones, see Fig. 1. The internal structure characterizes both the mechanical and optical properties of the polymer material. Hence, the majority of engineering polymers have a semi-crystalline structure; however, these polymers usually exhibit weak optical transmittance.

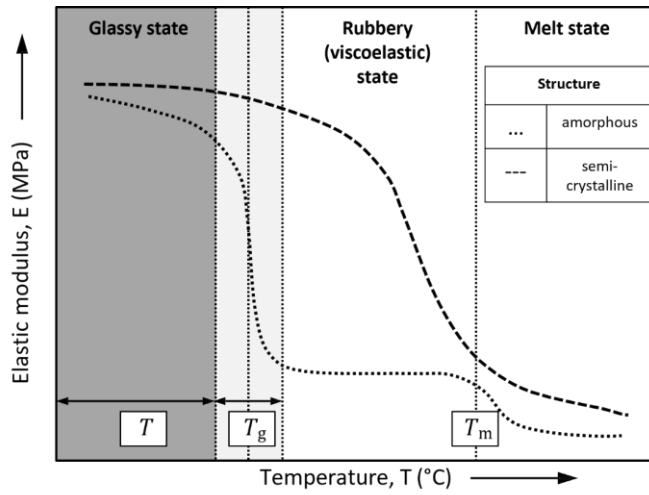


Fig. 1. Temperature dependence of elastic modulus E for the amorphous and semi-crystalline polymers.

In addition, the constitutive temperature-dependent behavior of polymers is characterized by the relaxation transitions labeled alfa, beta, gamma and delta in a decreasing order to the temperature at which the main-chain and individual side-chains of the polymer internal structure are rearranged, and the material exhibits a different viscoelastic response [24]. The primary alfa-relaxation refers to the T_g of the polymer in the viscoelastic state when the motion of the main-chain (chain backbone) molecules is enhanced [25],[26]. However, beta, gamma and delta—secondary relaxations occur in the interval below T_g in the glassy state and result mainly from the molecular motions of the side-chains of the polymer [27] as well as heteroatom motions in the main-chain [28]. Simultaneously, the mobility of the main-chain molecules decreases relative to the alfa-relaxation state but does not fully vanish [29]. The internal energy for initialization of the primary or secondary relaxation transitions of molecules corresponds to the viscoelastic activation energy (E_A) according to the inverse alphabetical order, i.e., $E_{A, \text{beta}} < E_{A, \text{alfa}}$ [30].

In practice, the moduli E' and E'' are often determined experimentally by the creep and relaxation tests using the nanoindentation [18],[31] or by a dynamic mechanical analysis (DMA) [32],[33] where the δ represents the phase angle between stress and strain corresponding to the interval of $\delta \in \langle 0, \pi/2 \rangle$. As the value of $\tan \delta(f, T)$ increases, the polymer exhibits more compliance (inverse parameter of stiffness) under mechanical cyclic loading, and thus, the material capability to damp the impacts and vibrations reaches a maximum [34].

The influence of time (or frequency f) of loading and temperature T can be combined to obtain the complex material response of the polymers by the time-temperature superposition principle (TTSP) [35],[36]. In this context, the Arrhenius (Eq. (3)) and the William-Landel-Ferry (WLF) (Eq. (4)) equations are mainly used in the temperature interval $T < T_g$ and $T \geq T_g$, respectively, to determine the time-temperature transformation by horizontal (a_T) and vertical (b_T) shift factors [27],[30]. This is associated with the selected reference temperature (T_R).

Arrhenius equation:

$$\ln a_T = \frac{E_A}{R} \left(\frac{1}{T} - \frac{1}{T_R} \right) \quad (3)$$

WLF equation:

$$\log a_T = -\frac{C_1 (T - T_R)}{C_2 + (T - T_R)} \quad (4)$$

where C_1 , C_2 are constants, for WLF, appropriate to the individual polymers, and R ($8.164 \text{ J mol}^{-1} \text{ K}^{-1}$) is a universal gas constant.

When the influence of time (or frequency) and temperature on the E^* is reciprocally equivalent, the polymer is considered to be thermorheologically simple, covering a single relaxation time spectra $H(\tau)$ [35], where τ is the relaxation time. Thus, the viscoelastic deformation is governed only by a single molecular process. Consequently, the single master-curve (MC) of the E^* or $\tan \delta$ covering the wide temperature and time (or frequency) interval can be derived for the individual thermorheologically simple polymers by a_T . This provides the necessary input data to include the viscoelastic response of the material in the tribological models. Otherwise, the polymer is thermorheologically complex [37] and the TTSP is not applicable [29].

Common failure modes of the polymer gears include fatigue, melting wear, plastic deformation, creep, and pitting [10],[38],[39]. In dry conditions, polymer gears often operated only at low loads and speeds. However, at higher speeds and loads, the friction forces increase resulting in a significant thermal softening effect, especially near the surface of the teeth flank. The consequence of this phenomenon is increased wear, which, in the extreme cases, can lead to melting wear [40]. Until recently, the polymer gears were frequently operated in low performance applications under dry conditions [8] without grease or liquid lubricant. The operating temperature (referred to as the ambient temperature, T) for lubricated polymers can exceed 150°C , which is significantly higher than the T for polymers operating under dry conditions.

The T can differ significantly from the bulk temperature of the gear and from the flash temperature in the contact [41],[42], as was demonstrated by Mao et al. [43],[44] under dry conditions, where a difference of about 20°C was observed between the point on the contact tooth and the point on the non-contact tooth. The flash temperature corresponds to the temperature at the contact region during the meshing cycle. In the interference of gears, the heat generation is influenced by viscoelastic behavior (hysteresis losses [45]-[48]) and frictional heat. For the latter, a significant effect was documented [9]. According to [49], the temperature increase in the contact between the steel/polymer gearset under lubricated conditions was noticeable, but not sufficient to cause a significant change in the mechanical properties of the polymer material and to affect the lubrication process.

In terms of kinematic conditions, when the teeth are in mesh, the sliding/rolling ratio (SRR) changes due to the movement of the pitch point along the pressure line [50]. Theoretically, pure rolling (SRR = 0) occurs only at the pitch point. As the sliding speed increases, the shear stress in the lubricant increases and the film thickness between the rubbing surfaces is reduced [10]. Under pure rolling conditions, the topography of the polymer body leads to smoothing of surface asperities [45],[46] in the contact area and thus to a reduction of sliding resistance. As a result, the surface roughness (R_q) is reduced, which affects the formation of the fluid-film thickness between the flanks of the teeth. In addition to T and SRR, the deformation characteristics of polymers are strongly dependent on the load (W), as was documented in [20],[40],[43], where the contact area was significantly enlarged.

Therefore, for the use of polymer gears in high-performance applications, it is necessary to ensure their operation in the liquid lubrication regime. The lubricant dissipates the frictional heat and maintains a coherent film thickness to ensure a full separation of the rubbing surfaces. In order to verify the existing theory and knowledge in the field of polymer lubrication, the identification of the lubrication regime is crucial.

2.2 Lubrication regimes, EHL modes, and lubricant rheology

One of the milestones in the lubrication science of the non-conformal contacts is Martin's [51] lubrication analysis in line contact of spur gears published in 1916. The individual lubrication regimes are generally characterized by the Stribeck's curve (see Fig. 2) which describes the dependence of the coefficient of friction (CoF) on the lubrication parameter (Λ) [52]-[54]. This parameter represents a relationship between the R_q , and the minimum film thickness (h_m) formed in the contact, see Eq. (5).

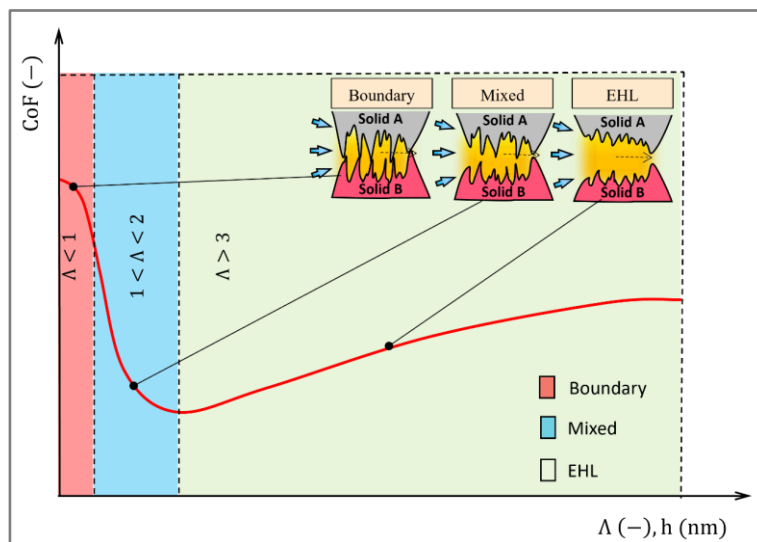


Fig. 2. Stribeck's curve and lubrication regimes.

Lubrication parameter Λ :

$$\Lambda = \frac{h_m}{\sqrt{R_{q1}^2 + R_{q2}^2}} \quad (5)$$

The development of the CoF in individual lubrication regimes results from a change in the ratio of the frictional components initiating from the direct contact between the surface asperities and the hydrodynamic friction in the liquid involving the rolling (Poiseuille flow) and sliding (Couette flow) frictional components [55]. With respect to the rheology of the liquid, the main properties of the lubricant are the dynamic viscosity (η) and the pressure-viscosity coefficient (α). The constitutive response of the material corresponds to the elastic state, assumed according to the Hertzian theory [56], where the Hertzian radius (a_H) for circular contact and the Hertzian pressure (p_H) are described by Eqs. (6, 7).

Hertzian contact radius a_H :

$$a_H = \sqrt[3]{\frac{3WR'}{2E_R}} \quad (6)$$

Maximum Hertzian pressure p_H :

$$p_H = \frac{3W}{2\pi a_H^2} \quad (7)$$

Where: R' - reduced radius of curvature $\left(\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}\right)$, E_R - reduced elastic modulus $\left(\frac{2}{E_R} = \frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2}\right)$

The first regime is the boundary lubrication regime where the direct full interaction of the surface asperities of the contacting solids occurs, and the rubbing surfaces are not separated by the film thickness, corresponding to the state when $R_q > h_m$. The second regime is the mixed lubrication regime, where partial contact of the surface asperities occurs, and the contact load is therefore transmitted by the surface asperities and, simultaneously, by the film thickness when $R_q \approx h_m$, [57],[58]. The mixed regime overlaps a transition from the boundary to the elastohydrodynamic lubrication (EHL) regime.

The EHL regime [59]-[69] is generally described by a full-film separation of the contact surfaces, preventing for the reciprocal contact of surface asperities $R_q < h_m$ [70]-[72]. This regime is typical of highly loaded non-conformal contacts, where the rubbing surfaces mesh over a very small area. In EHL, four significant lubrication modes can be distinguished according to the dependence on elastic deformations and the dependence of η on the contact pressure (p): Isoviscous-rigid (I-R), Piezoviscous-rigid (P-R), Isoviscous-elastic (I-E) and Piezoviscous-elastic (P-E) as was suggested by Johnson [73] using hydrodynamic maps of EHL, see Fig. 3.

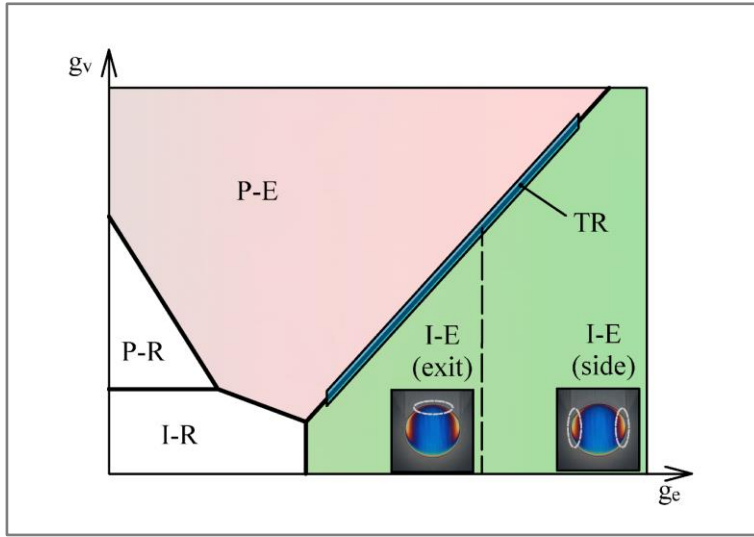


Fig. 3. Hydrodynamic map of elastohydrodynamic lubrication modes.

Classical EHL theory [60],[64],[66],[67],[69] is characterized by three crucial mechanisms influencing the formation of the central (h_c) and h_m film thicknesses in the contact. The first two mechanisms cover the pressure–viscosity effect [74]–[76] and the hydrodynamic effect of the fluid according to the Reynolds equation [77],[78] and thus represent the rheological response of the lubricant to the operating conditions. These mechanisms usually have a positive impact on the increase in fluid-film thickness in the EHL. However, the formation of film thickness in the EHL is strongly dependent on the conditions in the inlet region of the contact as Fig. 4 illustrated for circular contact (disc / ball).

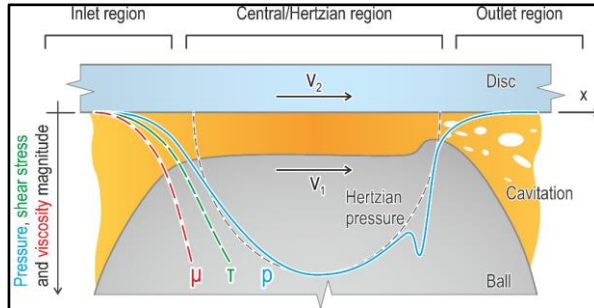


Fig. 4. Individual regions in the EHL contact.

The third mechanism is manifested by the constitutive material response of the solids, which commonly results in the elastic deformation of the rubbing surfaces after the non-conformal contact is loaded [15],[79]. This is typical of the P-E mode of (hard) EHL where the elastic deformation of rigid solids, such as metals, ceramics or glass, leads to high contact pressures (p) up to several gigapascals ($p \approx 3\text{--}4\text{ GPa}$) [62],[80]. In addition, the P-E mode is favored in the operation of steel bearings, cams and gears [55],[70] where the rheological behavior of the liquids has a significant influence on the formation of fluid-film thickness [81].

In terms of liquid rheology, the best-known pressure-viscosity formula ($\eta(p) = \eta_0 \exp(\alpha p)$) was derived by Barus [82]. Since then, Murnaghan [83], McEwen [84], Chu et al. [85], Roelands et al. [62],[63], and Jones et al. [86] have developed models that more accurately simulate the pressure-viscosity response, especially at high contact pressures ($p > 100$ MPa) and shear rates characteristic of the P-E mode of EHL. In addition, Doolittle [87] derived the relative volume model (RV) covering the pressure-viscosity and temperature-viscosity functions.

This allowed the rheological behavior of liquids to be simulated on a purely physical basis as was experimentally validated by Bair et al. [88]-[90] for several reference liquids with different compressibility using medium and high-pressure viscometers. Relative to this, the reference lubricants such as TOTM, SQL, 5P4E or PAO are often used because of their known properties covering temperature, pressure, and shear dependencies on the viscosity as shown by Bair et al. [88]-[90]. These lubricants are model liquids associated with the basic research and are often used especially in the field of the EHL and the fluid rheology. Lubricants with low compressibility may be more susceptible to the initiation of stresses in the contacting solids, which may result in surface fatigue.

Pressure-viscosity coefficient α_0 :

$$\alpha_0 = \left[\frac{d(\ln \eta)}{dp} \right]_{p=0} \quad (8)$$

Asymptotic Isoviscous pressure p_{iv} :

$$p_{iv} = \int_0^p \frac{\eta_0 dp}{\eta(p)} \quad (9)$$

In general, a pressure-viscosity response of the lubricant is described by the α coefficient, also called the initial pressure-viscosity coefficient α_0 , see Eq. (8). Another parameter, the asymptotic isoviscous pressure (p_{iv}), see Eq. (9), was defined as a lubricant property according to the Roelands et al. [62],[63], and is the inverse parameter to the reciprocal isoviscous pressure-viscosity coefficient (α^*), see Eq. (10). In hard EHL, the α^* was then used by Hamrock & Dowson [91] in fluid-film thickness calculations.

Pressure-viscosity coefficient α^* :

$$\alpha^* = \frac{1}{p_{iv}^\infty} = \left[\int_0^\infty \frac{\eta_0 dp}{\eta(p)} \right]^{-1} \quad (10)$$

Pressure-viscosity coefficient α_{film} :

$$\alpha_{film} = \frac{1 - \exp(-3)}{p_{iv}(3/\alpha^*)} \quad (11)$$

The last parameter is the pressure-viscosity coefficient (α_{film}) proposed by Bair et al. [88]-[90], see Eq. (11), which allows to directly compare the formation of the film thickness of different liquids with the same compressibility, without depending on the function $\eta(p)$ that is characteristic of each liquid. According to the McEwen model [84], Eqs. (8, 11) can alternatively be described by Eqs. (12-14) as shown by Bair et al. [90] by reducing the model parameters (then η_0 , α^* , q where the η_0 - dynamic viscosity at atmospheric pressure and q - McEwen coefficient).

Dynamic viscosity η :

$$\eta = \eta_0 \left(1 + \frac{\alpha^*}{q-1} p \right)^q ; q > 1 \quad (12)$$

Pressure-viscosity coefficient α_0 :

$$\alpha_0 = \frac{\alpha^*}{\left(1 - \frac{1}{q} \right)} \quad (13)$$

Pressure-viscosity coefficient α_{film} :

$$\alpha_{\text{film}} = \frac{1 - \exp(-3)}{1 - \left(1 + \frac{3}{q-1} \right)^{1-q}} \quad (14)$$

Nowadays, especially for the polymer machine elements, there are efforts to operate them in the EHL regime [10],[73],[92], due to the absence of friction between surface asperities, and the minimized generation of the friction in the liquid relative to a thin coherent fluid-film thickness. However, due to the low stiffness of the polymers compared to metals, the operating conditions result in a shift of the contact closer to the I-E mode of (soft) EHL.

The I-E mode occurs between non-conformal surfaces when the p is sufficient to induce elastic deformation of one or both rubbing surfaces, but is insufficient to induce a change in η [73],[93]. Therefore, the fluid-film thickness is less dependent on the η and more dependent on the material properties, which may exhibit a significant compliance of the interacting solids. Hence, the pressure-viscosity change is considered as insignificant in the I-E mode of EHL [81].

However, in the transition region (TR) between the P-E and I-E modes of EHL, this change could be expected in dependence on the material properties of the contact solids, the lubricant used and the operating conditions. In addition, the TR region [94]-[96] was described as the operating region of the polymer machine elements lubricated with mineral lubricants. With respect to this, the operation of the compliant contact in the TR region [95], [96] is assumed; however, only limited research in this field of EHL has been performed so far. Moreover, the prediction models of the film thickness corresponding to the P-E and I-E modes of EHL for the TR region have not been sufficiently validated yet. The analysis of the following studies describes the still insufficiently explored field of the soft EHL in the TR region and compares the results based on the numerical and experimental approaches.

2.3 Formation of film thickness in soft EHL contacts – numerical approach

Numerical studies represent the bulk of the state of the art in the field of soft EHL, which has often been neglected in the past compared to the hard EHL. This fact is related to the development of polymer materials and the possibility of their implementation in engineering applications over the last fifty years. The basis of numerical studies are usually Reynolds equations, elasticity equations and findings of authors historically dating back to the middle of the 20th century. Essentially, the outcome of these studies is the prediction of the contact pressure and film thickness under both static and dynamic conditions.

Johnson (1970) [73] contributed to the extension of the existing knowledge in the field of EHL for compliant contacts, following Archard [65] and Greenwood [67], whose numerical studies were aimed at determining the parameters describing the individual EHL modes. The formation of the fluid-film thickness is dominated by two main effects: an increase in viscosity with pressure and a constitutive material response when surfaces interact, leading to the elastic deformation of the rubbing surfaces. Both phenomena are typical of rigid contacts and correspond to the conditions of the P-E mode.

The result of the numerical study [73] is the determination of the dependence of the film thickness on the variation of the η and the influence of the elastic deformation of the contact surfaces. In this study, the relationships derived in studies [64]-[67] were compared. To separate the two main effects, the model proposed by Theyse [61] was used by defining the viscosity parameter g_v (also referred to as g_1) and the elasticity parameter g_E (also referred to as g_3), which correspond to Eqs. (15) and (16), respectively.

$$g_v = \left(\frac{\alpha^2 W^3}{\eta_0 U_E R'^2} \right)^{1/2} \quad (15)$$

$$g_E = \left(\frac{W^2}{\eta_0 U_E E_R R'} \right)^{1/2} \quad (16)$$

$$g_4 = \left(\frac{\alpha^4 U_E \eta_0 E_R^3}{R'} \right)^{1/4} \quad (17)$$

On this basis, the hydrodynamic maps of the EHL modes were derived. Except for the g_V and g_E parameters, the g_4 parameter (see Eq. (17)) was implemented as proposed by Archard [65] which characterizes the TR region. For this region, Johnson [73] derived an interval where $g_4 \in \langle 0.1, 2.2 \rangle$ based on the models published by Dowson & Higginson [60] and Herrebrugh [66].

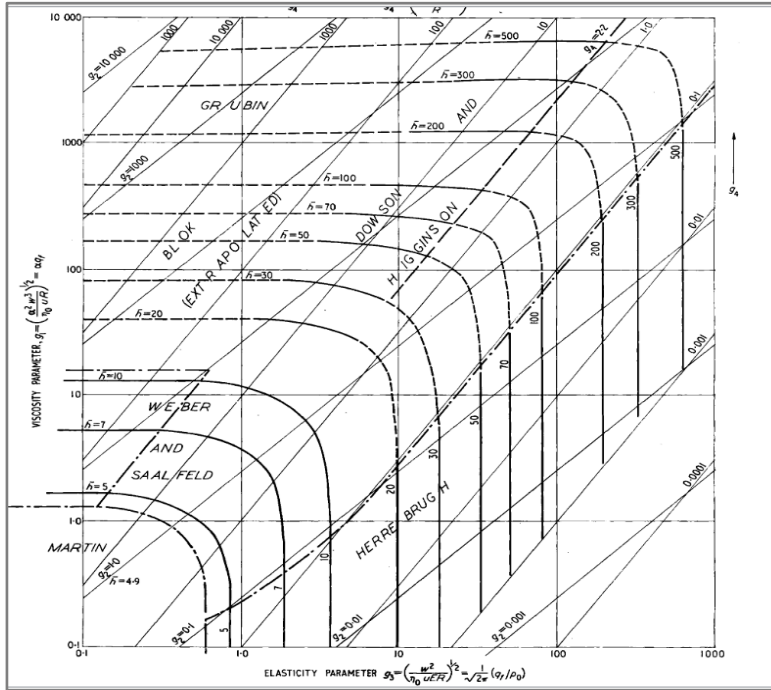


Fig. 5. Hydrodynamic map of EHL for circular contacts operating in I-E mode. [73]

The variation of the film thickness is well seen in the given interval of Fig. 5, where, at low values $g_4 \approx 0.1$, there is a significant dependence on the g_E . On the contrary, according to Fig. 5, as g_4 increases, there is only a dependence on the g_V . For $g_4 > 2.2$, there is an increase in η in the contact, accompanied by a sudden pressure change (pressure peak) at the exit of the lubricant from the contact. Johnson further states that the occurrence of a pressure peak occurs only when the film thickness is not significantly dependent on the elasticity of the two contact bodies. The outcome of this study [73] defines the hydrodynamic maps that identify the boundaries between the individual EHL modes using the parameters g_E , g_V and g_4 [64]-[67].

The studies published by Hamrock & Dowson (H&D) [91],[97], represent one of the milestones of the of soft EHL, while they themselves are based on the first findings of the EHL according to [63],[66],[68]. The authors deal with the distribution of the fluid-film thickness under isothermal conditions in elliptical and circular contacts with respect to the effect of the entrainment speed (U_E), the load (W), the ellipticity (k), and the α . For soft and hard EHL, the H&D derived the formulae for predicting of h_c and h_m in circular and elliptical compliant contacts.

In these formulae, the effect of individual operating conditions is described by power exponents of the dimensionless parameters $\bar{U}$, $\bar{W}$, $\bar{G}$ and k . However, the material parameter $\bar{G}$ ($E_R \alpha$) should not have a significant effect on the formation of the film thickness [73] due to the low stiffness of the compliant contact bodies. Hence, the $\bar{G}$ parameter was omitted in the prediction formulae of the film thickness in I-E mode [91], in contrast to the P-E mode.

It was found that at higher speeds, the pressure increase at the inlet of the contact is higher than at low speeds. Compared to the hard EHL, the pressure spike at the exit from the contact was not observed, which is often attributed to the pressure-viscosity effect of the liquid, causing the increase in the fluid-film thickness. The h_m was identified in the horseshoe shape at the exit of the lubricant from the contact (h_{mr}).

The influence of the $\bar{U}$, according to the study [91],[97], was determined to be dominant, especially in the case of h_m , similar to hard EHL contacts. The other result of the study [91] is the determination of the dependence of the fluid-film thickness on the $\bar{W}$. The values of the $\bar{W}$ varied by up to an order of magnitude, while a significant change in the pressure profile and only a slight change in the film thickness profile were recorded.

In addition, the influence of the k on the h_m and h_c was described. With the increase of k , the film thickness increases, but the results of the numerical solution differ from the EHL theory [63],[66],[68]. Prediction formulae of h_c and h_m in soft elliptical contacts are given in Eqs. (18, 19), where very close dependencies of the exponents of k , $\bar{U}$, and $\bar{W}$ for h_m and h_c were observed. For comparison, Eqs. (20, 21) are also derived by the H&D [97] for hard EHL contacts.

I-E EHL:

$$H_c = \frac{h_c}{R'} = 7.32 (1 - 0.72 e^{-0.28 \cdot k}) \bar{U}^{0.64} \bar{W}^{-0.22} \quad (18)$$

$$H_{mr} = \frac{h_{mr}}{R'} = 7.43 (1 - 0.85 e^{-0.31 \cdot k}) \bar{U}^{0.65} \bar{W}^{-0.21} \quad (19)$$

P-E EHL:

$$H_c = \frac{h_c}{R'} = 2.69 (1 - 0.61 e^{-0.73 \cdot k}) \bar{U}^{0.67} \bar{W}^{-0.067} \bar{G}^{0.53} \quad (20)$$

$$H_{mr} = \frac{h_{mr}}{R'} = 3.63 (1 - e^{-0.68 \cdot k}) \bar{U}^{0.68} \bar{W}^{-0.073} \bar{G}^{0.49} \quad (21)$$

Hooke [93],[98]-[100] began studying the behavior and numerical modeling of lubricated soft contacts in the early 1970s. He followed up on the theory and earlier numerical studies published by Blok [41], Dowson & Higginson [60] Archard [65], and Herrebrugh [66]. In the first mentioned study, Hooke [93] described the modeling of the pressure and film thickness distribution in soft EHL contacts. In the second [98] and third [100] studies, a numerical model was developed to calculate the h_m and h_c . This model describes the operating conditions by the pressure (P) and speed (S) parameters introduced by Greenwood [67] and by the dimensionless parameters k , $\bar{U}$, and $\bar{W}$.

Hooke [100] also describes the transitions between EHL modes, in particular between I-R and I-E. In both modes, there is no piezo-viscous effect due to pressure changes at the lubricant inlet and outlet from the contact, as is the case between P-E and P-R modes. The relocation of the h_m between the exit and the side lobes of the contact was also described, depending on the operating conditions, corresponding to the individual EHL modes [100].

In the case of the pressure distribution, a parabolic trend was observed in the outlet region of the contact, as in the case of the Hertzian dry contact [56]. Based on the numerical solution of the pressure distribution, the distribution of the fluid-film thickness was determined for different contact geometries. For the circular contact ($k = 1$), the formulae for h_c , h_{mr} , and h_{ms} are given in Eqs. (22-24). In addition, the constants necessary to determine the pressure distribution are given in [93].

I-E EHL:

$$H_c = \frac{h_c}{R'} = 3.56 \bar{U}^{0.6} \bar{W}^{-0.13} \quad (22)$$

$$H_{mr} = \frac{h_{mr}}{R'} = 2.83 \bar{U}^{0.6} \bar{W}^{-0.13} \quad (23)$$

$$H_{ms} = \frac{h_{ms}}{R'} = 8.55 \bar{U}^{0.75} \bar{W}^{-0.23} \quad (24)$$

In the study [100], the relationships between the boundaries of the I-R and I-E modes of EHL were identified. In addition, the effect of the ellipticity parameter k on the distribution of film thickness in narrow elliptical ($k < 1$), wide elliptical ($k > 1$) and circular contact ($k = 1$) was described. The transition from I-R to I-E mode was recorded at low values of the S parameter. In the I-E mode, due to the low values of P and S parameters, the h_m is localized at the exit from the contact, while, with increasing P and S , the h_m was identified at the side lobes of the contact. This transition was later observed in the experimental study of Marx et al. with the increasing W .

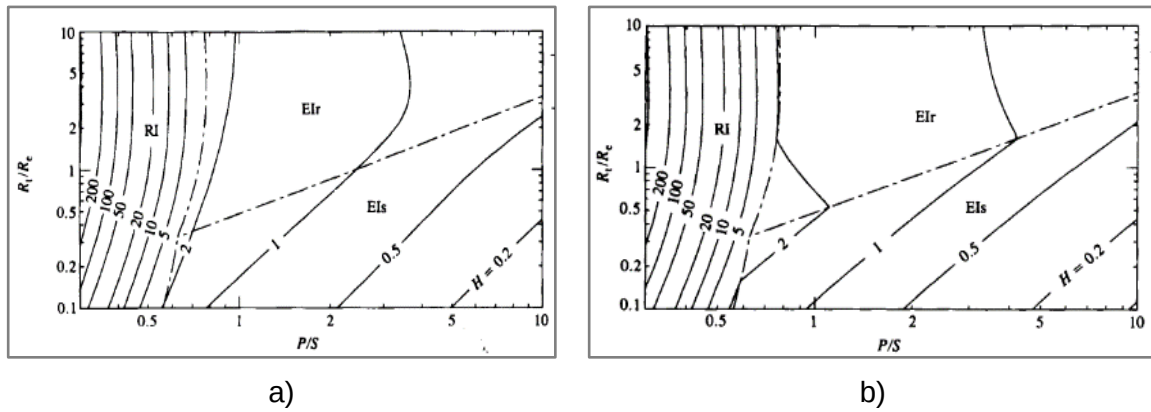


Fig. 6. Hydrodynamic map for circular contacts covering I-E and I-R modes. a) Johnson [73], b) Hooke [100]

The second transition region is the boundary between the I-E and P-E modes. The initialization of the transition depends mainly on the S parameter since the film thickness is already formed at the inlet to the contact and therefore does not depend on the maximum Hertzian pressure [100]. In the case of narrow elliptical contacts, no transition was observed between P-E rear and I-E rear, but directly to of I-E side, or P-E rear.

Thanks to the elasticity parameter (P/S), individual EHL modes can be distinguished when, at low values of P/S , the contact operates in the I-R, see Fig. 6, where the h_m is strongly dependent on the W . Conversely, at higher values of P/S in elastic modes of EHL, a dependence on the W is negligible. As the k increases, the transition of the h_m to the exit of the contact was recorded. The comparison of the hydrodynamic maps given by Johnson [73] in Fig. 6a and Hooke [100] in Fig. 6b shows the absence of sharp boundaries between individual modes.

Figure 6 shows a contour plot of the of film thickness for elliptical and circular contacts at different P/S ratios. Even at the lowest P/S , the h_m is formed at the contact exit. Figure 6 also shows that a horseshoe shaped film is formed as the P/S increases. The h_m was identified at the contact exit when $k = 2$. For circular ($k = 1$) and narrow elliptical ($k = 0.5$) contacts, the h_m was identified at the side lobes of the contact.

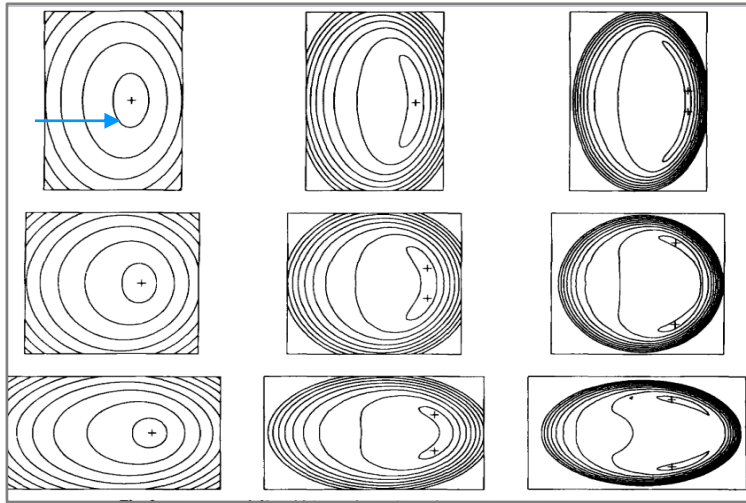


Fig. 7. Contour plot of film thickness for circular and elliptical contacts and different P/S ratios. ($k = 2, 1$ and 0.5 , and $P/S=0.78; 1.14$ and 1.67). Lubricant inlet from the left (see arrow). [100]

This trend can be seen in the further increase of the P/S ratio. In addition, for $k = 0.5$, a shift of the minimum towards the central region of the contact was observed. As the value of P/S increases, a partial disappearance of the horseshoe shape occurs where the presence of segments of the side lobes is shown. This behavior is typical for highly loaded contacts when U_E is applied along the main axis of the contact [100]. Hooke states that, for $P \geq 1$, the h_m occurs at the side lobes of the contact when $k \leq 1$.

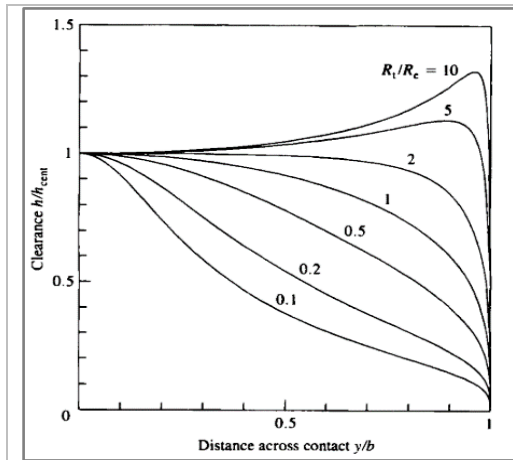


Fig. 8. Dimensionless film thickness vs. distance across the contact for highly loaded compliant circular contacts. [100]

Next, the dependence of the dimensionless thickness (H) on the k for highly loaded point contacts was also described, see Fig. 8. For the $k \leq 2$, the results showed a gradual decrease of H from the contact center to the side lobes. For $k > 2$, a short-term increase in H was observed before reaching the area of the side lobes, as shown in Fig. 8.

Based on the [100], the fluid-film thickness in elliptical and circular soft EHL contacts can be determined by converting the P and S parameters to the $\bar{U}$ and $\bar{W}$ parameters. The film thickness is then predicted similarly as for circular contacts, see Eqs. (22-24). These results extend the state of the art on the formation of film thickness in line, circular, and elliptical compliant contacts and allow for a comparison of a numerical model with experimental data, similar to H&D [91].

In another study, Esfahanian [96] described individual EHL regimes and established the formulae for predicting h_m . The study extends the knowledge of soft and hard EHL regimes published by Johnson [73], Hamrock & Dowson [101], and Bergner [102]. Esfahanian gives the relationships $h = f(H, k, \bar{U}, \bar{W}, \bar{G})$, which are used in the calculation of the film thickness and represent the dependence on the operating conditions in the EHL contact.

Due to the separation of the two main effects in the formation of the film thickness in EHL contacts, a change in η and elastic deformation, the parameters g_V and g_E were used instead of the above dependencies, similar to the study by Johnson [73]. This reduced the model parameters to $h = f(\hat{H}, k, g_V, g_E)$. Other dependencies were described by Eqs. (25, 26). According to g_V and g_E , hydrodynamic maps for circular and elliptical contacts were composed to represent the individual EHL modes together with their transition regions in a similar manner, as was suggested by Johnson [73]. In addition, Esfahanian [96] proposed new prediction formulae of h_m for the I-E mode by Eq. (27) and for the P-E mode by Eq. (28).

Dimensionless film thickness parameter $\hat{H}$:

$$\hat{H} = H \left(\frac{\bar{W}}{\bar{U}} \right)^2 \quad (25)$$

Ellipticity parameter k :

$$k = \frac{R_y}{R_x} \frac{2}{\pi} \left(\frac{R_y}{R_x} \right)^{2/\pi} \quad (26)$$

I-E EHL:

$$H_m = \frac{h_m}{R'} = 8.7 g_E^{0.67} (1 - 0.85 e^{-0.31k}) \quad (27)$$

P-E EHL:

$$H_m = \frac{h_m}{R'} = 3.42 g_E^{0.17} g_V^{0.49} (1 - 0.85 e^{-0.684k}) \quad (28)$$

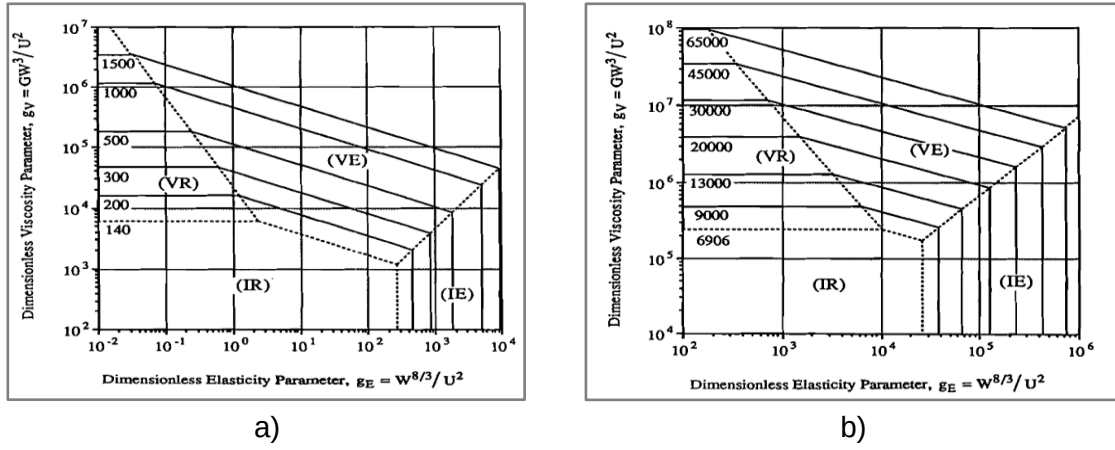


Fig. 9. Hydrodynamic map for different contact geometries. A) circular contact ($k = 1$), b) wide elliptical contact ($k = 6$). [96]

For the circular contact, the hydrodynamic map in Fig. 9a shows a shift toward the boundary with the I-E mode with the increasing g_E compared to a narrow elliptical contact. For a wide elliptical contact, see Fig. 9b, this boundary is shifted even further to the right. The development of g_V is similar to that of the g_E parameter which increases relative to the increase in k .

Myers et al. [95] investigated the distribution of fluid-film thickness and pressure in the TR region between the I-E and P-E modes of EHL in a line contact. The study extends the existing results published by Herrebrugh [66], Hall [69], Johnson [73], and Hooke [93], [98]. Myers et al. present the results of the numerical approach in Fig. 10, where the TR region is bounded by an interval of g_4 depending on the parameters g_V and g_E . In addition, the author states that the H_m did not fall below 4.9, which corresponds to the results given by Martin [51] for the I-R mode.

Based on the numerical model [95], the Eq. (29) and Eq. (30) were derived to calculate the H_m , valid for the interval of $g_E \in \langle 1, 100 \rangle$. Compared to the results published by Hooke and O'Donoghue [93], there is a good agreement in the interval of $20 < g_E < 100$. According to the numerical model, a linear dependence of the g_E on the H_m was obtained. The results obtained by Myers [95] for the I-E mode are close to those obtained by Blok [41] and Greenwood [67] for the I-R mode of EHL at an increase of g_4 .

$$H_m \in (1 < g_E \leq 15):$$

$$H_m = 0.383 g_V g_E^{-0.370} + 4.465 g_E^{-0.667} \quad (29)$$

$$H_m \in (15 < g_E \leq 100):$$

$$H_m = 0.853 g_V g_E^{-0.664} + 3.410 g_E^{-0.770} \quad (30)$$

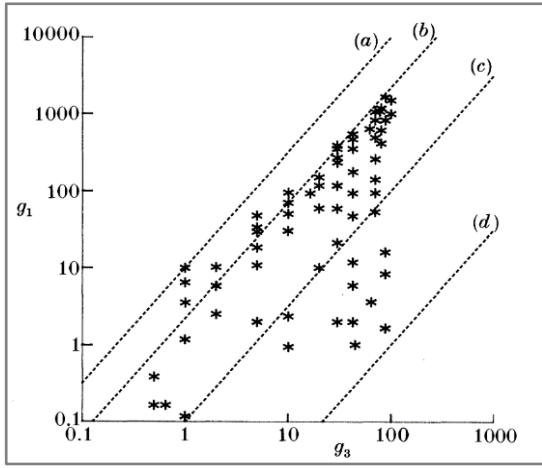


Fig. 10. Hydrodynamic map for the line contacts (according to Myers model). [95]

Hooke & Huang [103] extended the knowledge in the field of soft EHL by implementing the influence of the constitutive viscoelastic response of the material in the line contacts. This study broadens the findings of Johnson [73] and Hunter [104]. The dependencies of compliant materials, which exhibit a time delay in the deformation of the contact bodies, have been described. This affects the formation of the fluid-film thickness and the pressure distribution; however, these phenomena have not yet been sufficiently investigated experimentally. Based on the results of this study [103], the contact pressure and fluid-film thickness were described as a function of the Deborah number (D), see Eq. (31). For $D = 0$, the state corresponds to the purely elastic response of the material.

Deborah number D :

$$D = \frac{\tau U}{b} \quad (31)$$

where: τ - relaxation time of the polymer, U - surface speed of viscoelastic solids, and b - Hertzian contact semi-width.

The pressure profiles in Fig. 11a and Fig. 11b show the expected increase in p and a decrease in b with the increasing D . For the dry contact, a sharp increase in p at the contact inlet was documented, except for $D = 0$. The pressure distribution at low D is close to the development of the Hertzian pressure distribution, except for the inlet and exit regions of the contact. A constriction is formed at the contact exit, which is typical of hard EHL. Under lubricated conditions (see Fig. 11b), the slope of the pressure profile is evident when $D \in \langle 0.5, 2 \rangle$, with the maximum peak oriented towards the contact inlet which decreases in magnitude as D increases.

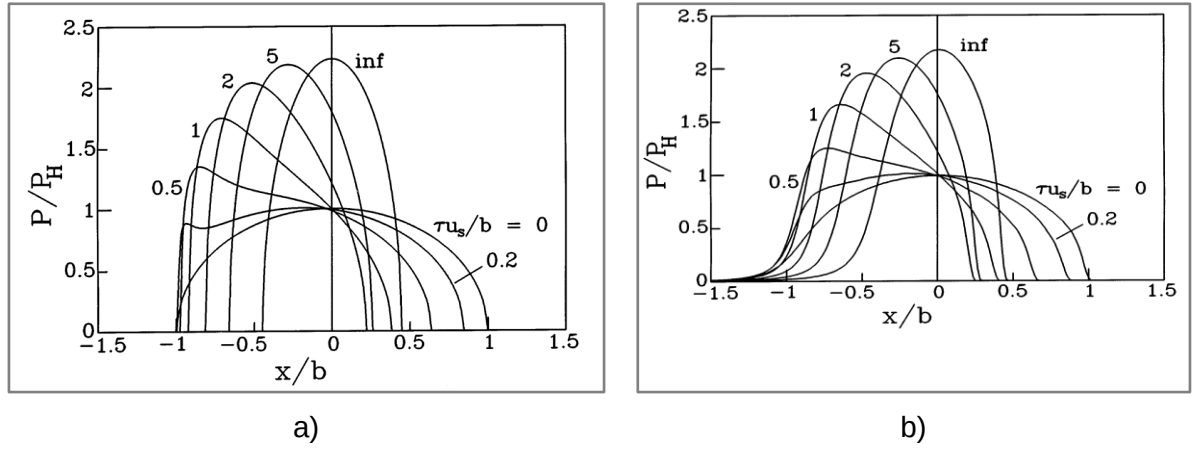


Fig. 11. Contact pressure profiles for viscoelastic contacts. a) dry conditions, b) lubricated conditions. [103]

For different D , the fluid-film thickness demonstrated an almost constant distribution over the central contact region, see Fig. 12. As soon as $D > 0$, there is an evident decrease of the fluid-film thickness. For the interval $D \in (0.5, 2)$, the film thickness is almost constant with the constriction at the contact exit and decreases with the increasing D with respect to the R_c and h_m . However, the film thickness profile is qualitatively very similar to that of the hard EHL. With increasing g_E , the film thickness and the length of the input and exit regions of the contact decrease, while the position of the minimum corresponds to the h_{mr} .

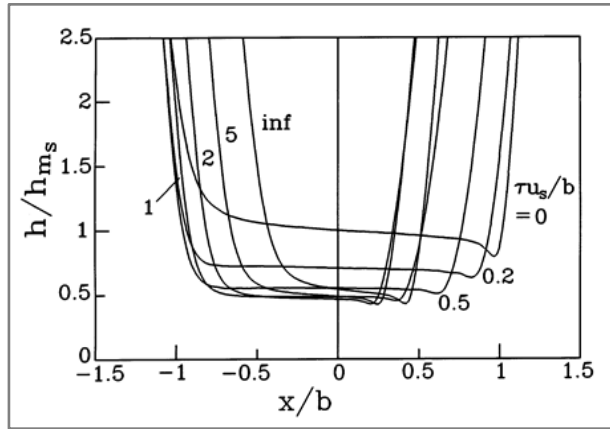


Fig. 12. Film thickness profiles for different Deborah's numbers. ($g_E = 30$) Lubricant inlet from left. [103]

Low values of D are associated with the similarity of the contact pressure distribution to the Hertzian model, which assumes the elastic response of the material. As the g_E increases, the constitutive material behavior changes from the elastic to the viscoelastic state. At high D , the influence of the viscoelasticity increases and then the fluid-film thickness decreases.

Furthermore, at low D , the pressure development at the inlet of the contact showed a steep increase compared to the exit of the contact. This influence is reflected in the film thickness profiles with a horizontal straightening of the profiles in the central region. The decisive criterion whether the compliant contact exhibits an elastic or viscoelastic response is the Deborah number, with the transition occurring at $D \sim 1$.

Stupkiewicz et al. [105] presented a numerical study aimed at determining the distribution of film thickness and pressure in the I-E mode of EHL between the NBR polymer ball and the low-carbon steel disc. The development of the film thickness was simulated for several W and the products of $U_E \eta$. The results showed that the formation of a characteristic “spine” on the trailing edge of the contact was noticeable.

With respect to the operating conditions, the minimum occurs at the exit of the contact as well as at the side lobes, see Fig. 13. The latter occurs especially with increasing W at low U_E in accordance with the results published by Hooke et al. [100]. At the same time, the contact area (A) was significantly enlarged with increasing W [105].

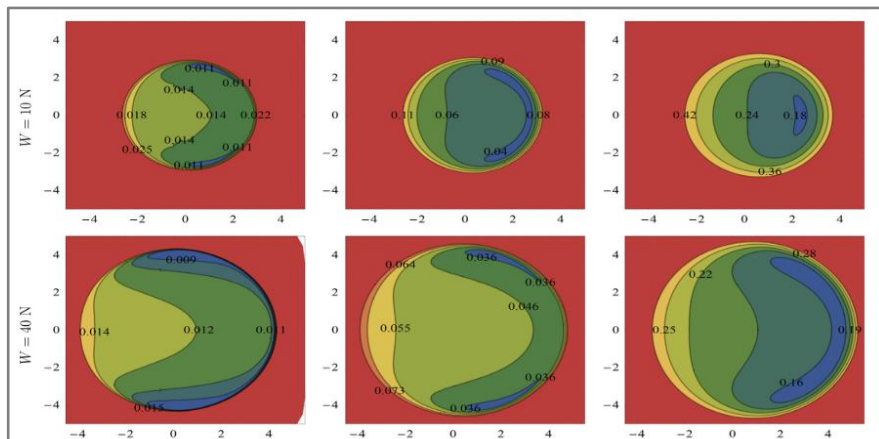


Fig. 13. Distribution of film thickness for different $U_E \eta$ ratios and loads W . Lubricant inlet from left. [105]

2.4 Formation of film thickness in soft EHL contacts – experimental approach

In recent years, a limited number of experimental studies have been published that extend the state of the art on the formation of film thickness in compliant contacts operating in the EHL regime. Thus, the issue of soft EHL has not been sufficiently explored experimentally yet. This is mainly due to the limitations of some experimental methods, especially optical methods, with respect to the low light transmittance of most crystalline and semi-crystalline engineering polymers.

Fluorescence, ultrasonic reflection or optical interferometry [106]-[108] methods are commonly used to measure the fluid-film thickness. The latter method has been widely used in the past, mainly for hard EHL. The necessary condition for the application of this method is the use of at least one transparent material to ensure the passage of light into the contact. The experimental data are then usually compared with numerical models and the EHL theory [91],[109]-[113]. Thus, new prediction formulae of both h_c and h_m can be derived based on the regression of the experimental data.

Myant et al. [114],[115] focused on the lubrication of soft EHL contacts under high SRR's. The influence of the SRR can be manifested by an increase in CoF and a decrease in fluid-film thickness. Based on the operating conditions, such as U_E , W and η , the dependence on the formation of the film thickness was described. Using the numerical approach introduced by de Vicente et al. [113], the pressure distribution in the contact was determined inversely. Optical monochromatic interferometry [106],[107] was implemented to determine the distribution of fluid-film thickness between a glass (BK7, [116]) disc and a steel ball coated with a thick layer of PDMS elastomer, representing a compliant contact. Experimental results [114] are compared with published studies [91],[113].

Furthermore, film thickness profiles were evaluated from monochromatic interferograms in the perpendicular (x) and parallel (y) directions relative to the U_E . In both profiles, the expected increase in the R_c with the increasing W was observed, while the increase in W leads to a significant decrease in the fluid-film thickness along the contact centerline. In the exit region of the contact, a constriction with a hydrodynamic wedge is formed, where the h_m is identified, see Fig. 14. Simultaneously, an increase of inclination in y-profile and a constriction due to the decreasing W were observed. Pressure profiles were obtained based on the findings of Cameron [57] and Vicente et al. [113] where these profiles demonstrate a significant dependence on the applied W .

Fig. 14 shows good agreement between the contact pressure distribution and Hertzian theory along the contact centerline with increasing W , as predicted by Hamrock & Dowson [91]. In contrast, at low W , the pressure peak is shifted closer to the exit of the contact. The film thickness profiles are not significantly affected for either load. Myant et al. [114] state that the minimum was identified in the exit region of the contact - h_{mr} .

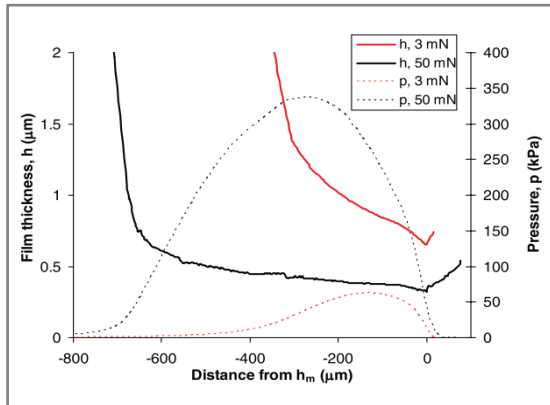


Fig. 14. Film thickness and pressure profiles. GLY lubricant. ($W = 3$ and 50 mN; $U_E = 0.65$ and 0.61 mm/s). [114]

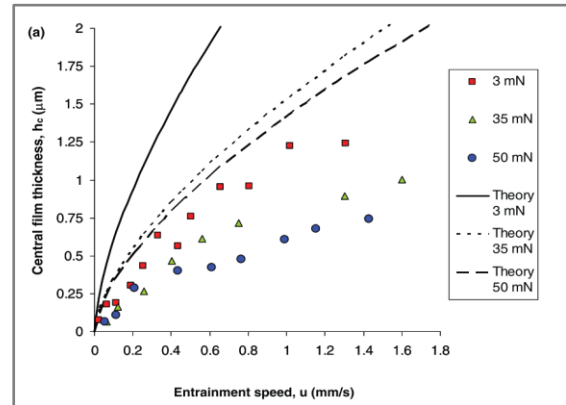


Fig. 15. Comparison of the experimental central film thickness with the Hamrock & Dowson model. GLY lubricant. ($W=3, 35, 50$ mN; $U_E=0-1,8$ mm/s). [114]

Compared to the EHL theory, the experimental results [114] manifested a significant deviation, see Fig. 15. Quantitatively, the predicted fluid-film thickness is several times higher than the experimental data. Due to the increasing U_E , the angle of inclination of the film thickness profile increases as well as the magnitude of the film thickness between the rubbing surfaces.

The effect of η on the film thickness profiles and pressure profiles is demonstrated in Fig. 16. When a more viscous lubricant is used, the p is reduced and the pressure peak is shifted to the vicinity of the center, similar to the Hertzian contact. The formation of a pressure peak in the profile, similar to the hard EHL, was not observed in the experiments. Based on the experimental data, Myant et al. [114] established the regression formulae for h_c and h_m , which are given in Eq. (32) and Eq. (33), respectively. The film thickness profiles in the [114] qualitatively corresponds well with the numerical model of de Vicente et al. [113]. However, the h_m qualitatively differs by up to 200 nm. Except for the centerline profiles, the distribution of the fluid-film thickness in the contact is also evident from the Fig. 17.

I-E EHL:

$$H_c = \frac{h_c}{R'} = 3.3 \bar{U}^{0.63} \bar{W}^{-0.13} \quad (32)$$

$$H_{mr} = \frac{h_{mr}}{R'} = 2.83 \bar{U}^{0.68} \bar{W}^{-0.20} \quad (33)$$

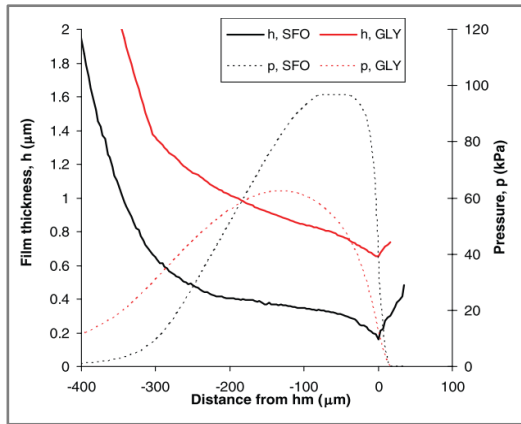


Fig. 16. Film thickness and pressure profiles. Lubricant SFO and GLY. $\eta_{SFO} < \eta_{GLY}$. ($W = 3$ mN; $U_E = 0.66$ mm/s). [114]

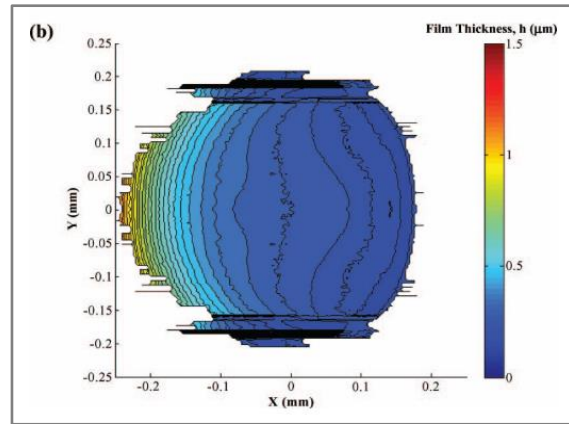


Fig. 17. Distribution of the film thickness in compliant contact. Lubricant GLY. ($W = 3$ mN; $U_E = 0.66$ mm/s). [114]

Gasni et al. [117] investigated the effect of W and SRR on the formation of film thickness in a circular compliant contact. The contact was formed between a nitrene ball ($E < 10$ MPa) and a Perspex glass disc. The experiments were performed in the load range up to 8 N, with the ball loaded in the normal direction. The contact is simulated in the static position of the disc, which, according to the EHL theory [52], corresponds to pure sliding when $SRR = 2$. In this study, the ultrasonic reflection method was used to determine the fluid film thickness, see Fig. 18 and Fig. 19.

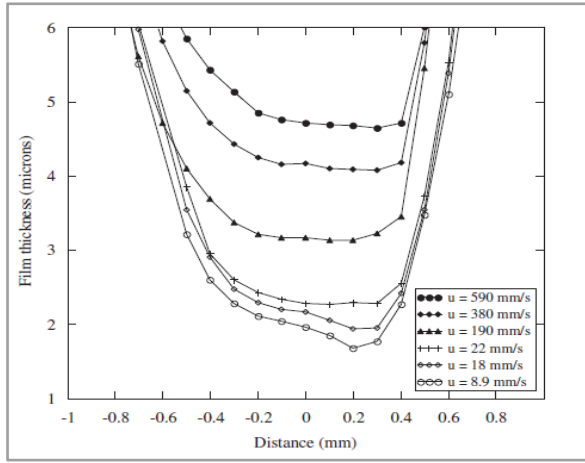


Fig. 18. Film thickness profiles vs distance as function of entrainment speed. ($W = 3.8\text{ N}$, $p \approx 1.04\text{ MPa}$). Lubricant inlet from the left. [117]

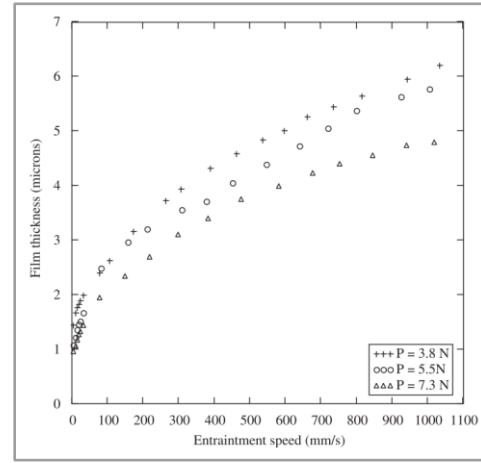
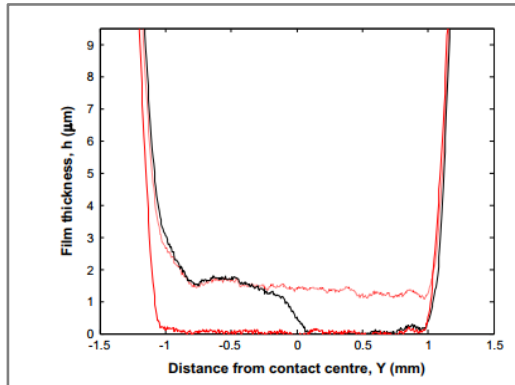


Fig. 19. Film thickness vs. entrainment speed as function of load. [117]

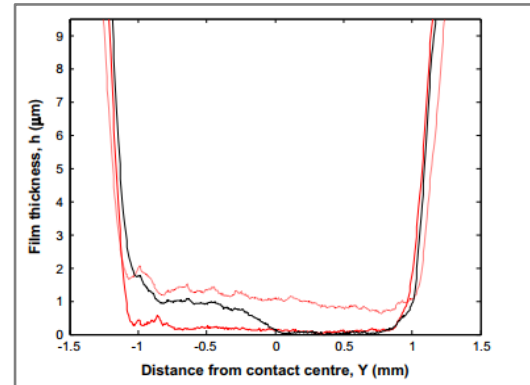
Gasni et al. [117] compared the film thickness with the H&D model [91] and the experimental results of Myant et al. [114],[115]. The measured film thickness was significantly lower than predicted by the H&D model, and this difference even increased with the U_E . Even at low U_E , a significant increase in p was observed, as predicted by de Vicente et al. [113].

The wedge-shaped profile can be seen in Fig. 18, at the inlet to the contact and the constriction at the exit of the contact, especially at low film thicknesses, as also found by Roberts et al. [109] and Myant et al. [114]. The elastic deformation of the rubbing surfaces is greater than the fluid-film thickness, so it can be assumed that the pressure distribution for the dry contact will be close to the Hertzian distribution. According to the parameters g_E and g_V introduced by Johnson [73], the contact was operated in the I-E mode of EHL [117].

Myant et al. [118] in this study aimed at the formation of fluid-film thickness for different kinematic conditions. The author used the Laser Induced Fluorescence (LIF, later used in [119]) method focusing on the change of the fluid-film thickness under transient motion conditions: start and sudden stop. The experiment was performed under pure sliding conditions ($SRR = 2$) for a static PDMS ball and a rotating glass (BK-7) disc.



a)



b)

Fig. 20. Film thickness profiles: static contact (red), stabilized lubricating film (dashed red), thickness course in the center of contact (black). Lubricant inlet from left. a) $1000 \text{ mm}\cdot\text{s}^{-2}$, b) $5 \text{ mm}\cdot\text{s}^{-2}$. [118]

In the inlet region of the contact, there are four phases of film thickness formation: initial contact deformation, separation of the surfaces, formation of a coherent film, and a stabilized coherent film thickness. When the lubricant is in contact ($t = 0 \text{ s}$), the contact profile corresponds to the Hertzian contact shape. For the higher of the two accelerations, the change in the film thickness development is more pronounced, while for the lower acceleration, it is a vice versa and the film thickness corresponds more closely to the steady film thickness profile, as shown in the comparison of Fig. 20a and Fig. 20b.

The second case in [118] is a sudden stop after the interval when a coherent film thickness is formed. The experiment shows the formation of a horseshoe shape with the h_m at the exit of the contact. The film thickness reaches a minimum at the edges of the contact, where the horseshoe shape moves toward the center of the contact with increasing time after a sudden stop. Therefore, even after a few seconds, a significant fluid-film thickness may still be present. The development of the film thickness for different sliding speeds is shown in Fig. 21a and Fig. 21b. It is evident that higher sliding speeds result in significant deviations in the film thickness profiles over time compared to low speeds.

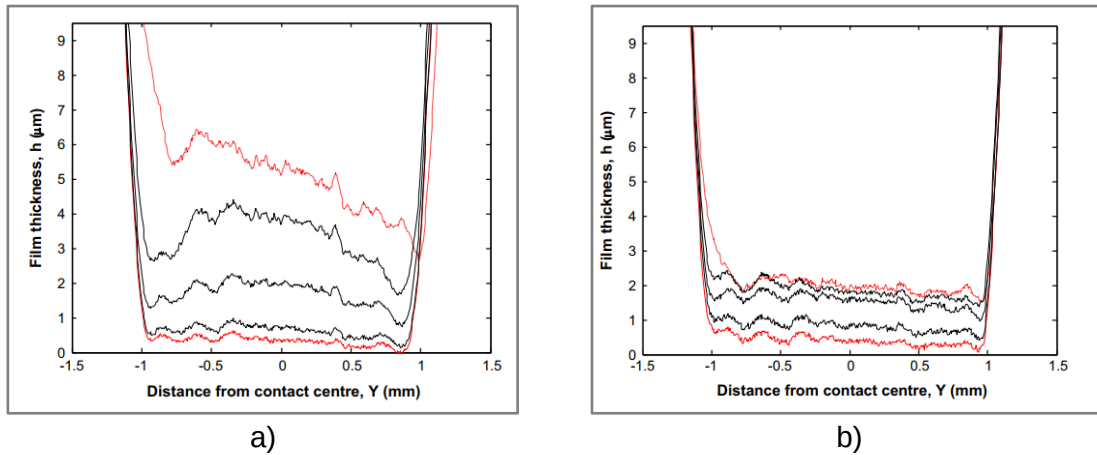


Fig. 21. Film thickness profiles at different sliding speeds: a) $75 \text{ mm}\cdot\text{s}^{-1}$, b) $10 \text{ mm}\cdot\text{s}^{-1}$. Lubricant inlet from the left. after $t = 5 \text{ s}$ (red), stabilized film thickness before sudden stop (dashed red), film thickness profiles at individual times t (black). [118]

Especially at lower accelerations, a gradual formation of the film thickness was observed. Myant et al. [118] state that the formation of the film thickness is strongly dependent on the magnitude of the initial acceleration. At low accelerations, there is a change in the film thickness beyond the center of the contact. Significant fluctuations in the film thickness profiles were not observed, which can be attributed to the nature of the soft EHL, as opposed to the hard EHL where this is different.

Dearn et al. [10] aimed at the formation of film thickness, development of working efficiency, and CoF between in the fully compliant and hybrid gearsets. In the study, the author determines the working efficiency of the gearset based on the pressure angle (α_G), W , speed, and E_R for the selected material combinations in the gearset. When determining the film thickness, the author refers to the results of Johnson's hydrodynamic maps [73] and the prediction formulae of the dimensionless film thickness according to Myers et al. [95] for the TR region between P-E and I-E modes of EHL. In order to implement the EHL theory, the author simplifies the complex gear mesh by the simulating the contact by two cylinders on the twin-disc apparatus, where a line contact is formed.

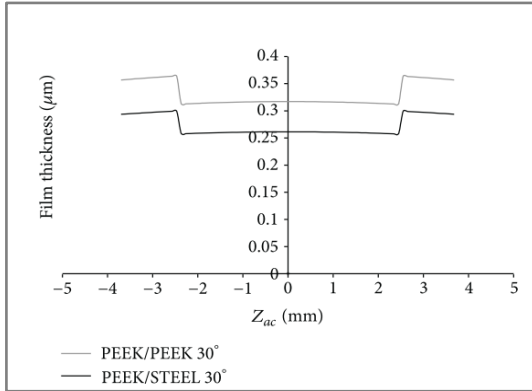


Fig. 22. Comparison of film thickness of different gearsets (PEEK/STEEL and PEEK/PEEK). [10]

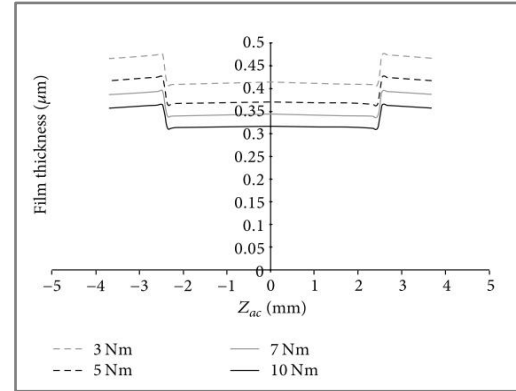


Fig. 23. Comparison of film thickness depending on load for combination (PEEK/PEEK). [10]

For the fully compliant the PEEK/PEEK gearset, a dependence on the pressure angle α_G was found; with the increasing α_G , the contact area and the film thickness are increased. At the same time, an increase in the rolling speed component was observed while the sliding component of the entrainment speed decreased. The results for the PEEK/PEEK showed that the film thickness is significantly higher than that for the hybrid PEEK/steel gearset, see Fig. 22, which is probably influenced by the elastic deformation with uniform load sharing for the former gearset. This agrees with the results published by Song et al. [120]. The effect of the W is shown in Fig. 23, which manifested the expected decrease in film thickness with increasing W . On the other hand, a significant increase in film thickness was observed with increasing the U_E .

The results of the efficiency measurement show the similarity between dry and lubricated conditions, where there is an increase in efficiency with increasing W compared to the results observed by Walton et al. [13]. At the same time, there is an elastic deformation of the tooth flank, which leads to a smoothing of the surface asperities and thus to a reduction of the CoF. The CoF was determined from the work efficiencies according to [13], which showed a dependence on W , while the effect of angular velocity was found to be insignificant.

Furthermore, no dependence of the operating efficiency and CoF on the pressure angle α_G was observed. However, the choice of the pinion and wheel gear material has a significant impact on the operating efficiency, as shown in Fig. 24a and Fig. 24b for PEEK/PEEK and PEEK/steel gearsets, respectively. Dearn et al. [10] attribute this phenomenon, to a greater extent, to the elastic deformation of the rubbing surfaces in the former gearset than in the latter gearset.

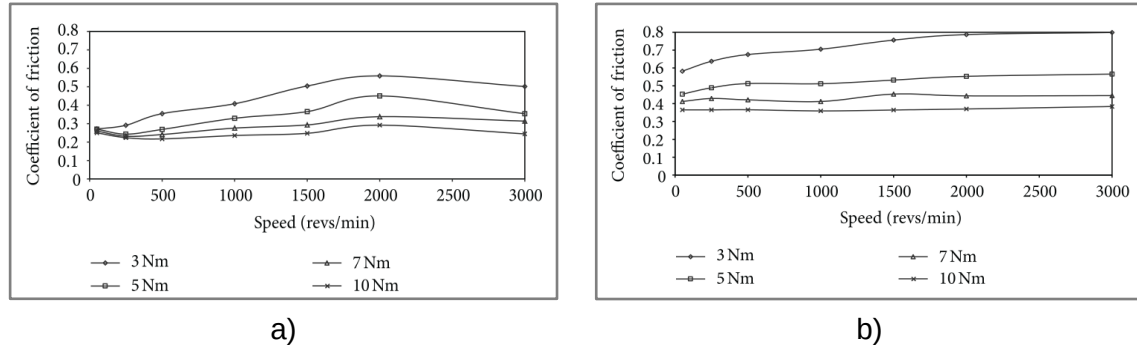


Fig. 24. CoF vs. angular velocity for different gearsets. a) PEEK/PEEK, b) PEEK/steel. [10]

Marx et al. [121] described the formation of the h_c and h_m in the circular compliant contact between the steel ball and the PMMA and PU discs operated in the EHL (same configuration used with greases in [122]). Optical chromatic interferometry was used to obtain the film thickness distribution in a compliant contact. This study focuses on the determination of the influence of U_E , SRR, W , and η on the film thickness under isothermal conditions.

The experimental results obtained were compared with the prediction models of soft EHL [91],[96], and [100]. Based on the regression model, the formulae of h_c and h_m were derived. During the increase of U_E under the pure rolling condition (SRR = 0), a horseshoe-shaped film is formed at the side lobes and at the exit of the contact, where h_m is identified, see Fig. 25. Except for the pure sliding conditions (SRR = 2), the h_m was always observed at the side lobes.

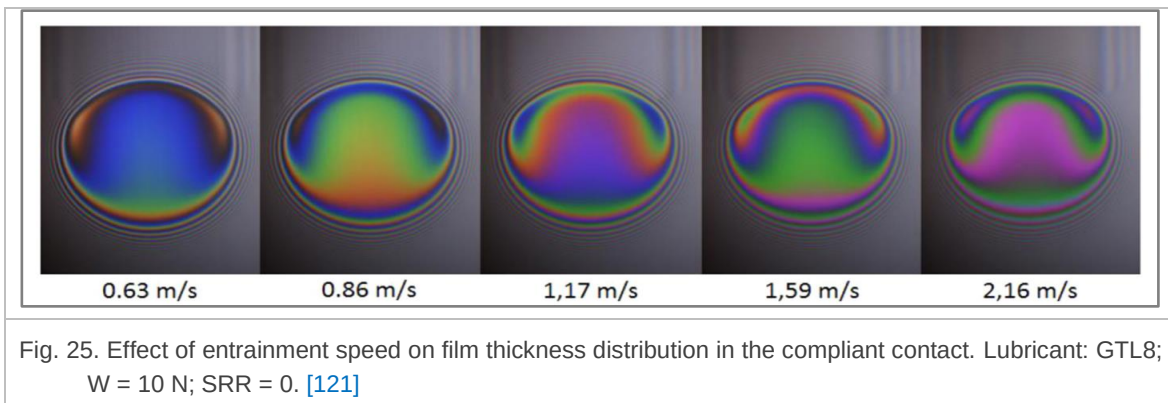


Fig. 25. Effect of entrainment speed on film thickness distribution in the compliant contact. Lubricant: GTL8; $W = 10$ N; SRR = 0. [121]

Regarding the film thickness profiles, an almost horizontal profile was recorded in the central region of the contact, which is typical especially for hard EHL contacts [31]. At the exit of the lubricant from the contact, there is a sudden decrease in the film thickness due to a wedging of the film thickness caused by a decrease in the p . A similar development was observed by Myant et al. [114].

Another set of experiments was aimed at interpreting the influence of SRR on the formation of the film thickness at a constant sliding speed, see Fig. 26. Except for $SRR = 2$, only a small change in film thickness was observed over the whole interval, where $SRR \in \langle -2, 2 \rangle$. This is interesting with respect to hard EHL, where this change was observed by Smeeth et al. [123] already at $SRR \geq 0.5$. Due to the high SRR, there is an increase in the temperature T in the contact, resulting in a decrease in η and thus in the film thickness.

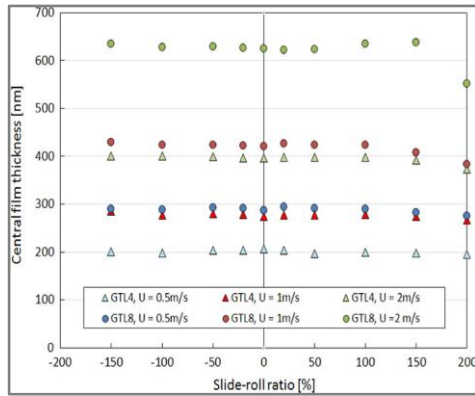


Fig. 26. The central film thickness h_c vs. different SRR ratios ($W=10$ N). [121]

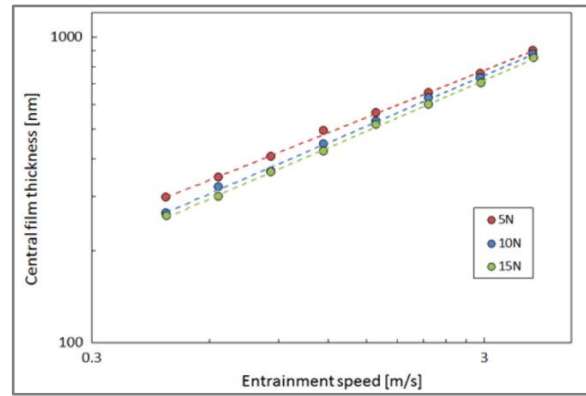


Fig. 27. The effect of the load on the central film thickness. ($SRR = 0$). [121]

The effect of the load ($W \in \langle 5, 15 \rangle$ N) on the h_c was insignificant, see Fig. 27. On the other hand, the transition of h_m from the exit region of the contact to the side lobes was recorded with increasing W . The contact area was also significantly enlarged with W due to the initiation of elastic deformation of the rubbing surfaces.

A significant deviation of h_{ms} and h_{mr} with U_E was also observed. At low U_E , there was a significant difference between h_{ms} and h_{mr} ; however, with increasing U_E , this difference gradually disappeared, see Fig. 28. Based on the experimental data, Marx et al. [121] derived new formulae for h_c and h_m , including the interval of the power exponent of the $\bar{U}$ as $x \in \langle 0.52, 0.55 \rangle$. However, these results differ from the prediction models of H&D [91], Esfahanian [96], and Hooke et al. [100], where $x \in \langle 0.6, 0.66 \rangle$.

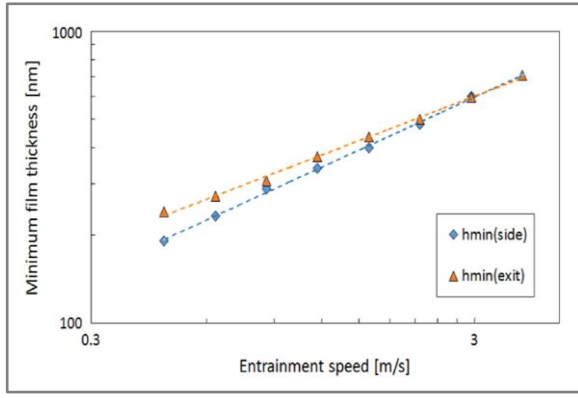


Fig. 28. Minimum film thickness vs. entrainment speed. (steel/PMMA; SRR = 0, W = 5 N). [121]

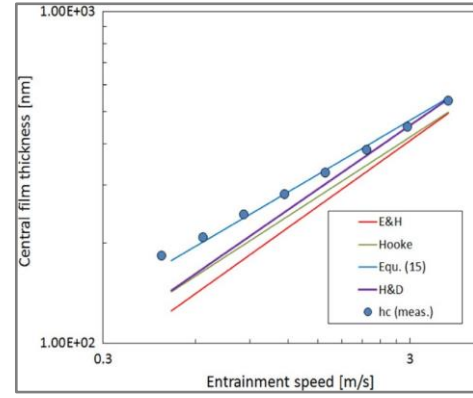


Fig. 29. Comparison of experimental data of central film thickness with soft EHL models. (SRR = 0). [121]

I-E EHL:

$$H_c = \frac{h_c}{R'} = 2.6 \bar{U}^{0.55} \bar{W}^{-0.07} \quad (34)$$

$$H_m = \frac{h_m}{R'} = 2.8 \bar{U}^{0.62} \bar{W}^{-0.16} \quad (35)$$

The prediction models [31],[96],[100] showed significant deviations from the experimental data [121], see Fig. 29, especially at low the U_E where the predicted h_c is significantly lower. The regressed formulae of the h_c and h_m are given in Eq. (34) and Eq. (35), respectively. These formulae can alternatively be expressed by the g_E according to Johnson [73], which is relevant for the demonstration of the results in the hydrodynamic map.

With respect to the current state of the art, only a few tribological studies [124]-[129] that dealing with the formation of fluid-film thickness in compliant contacts consider the influence of the constitutive viscoelastic response of the material. These studies usually combine the numerical modeling of the viscoelastic response together with the implementation of this response to the prediction models in soft EHL or with the experimental validation of the film thickness based on the numerical model used. However, the fully experimental studies dealing with the determination of the viscoelastic response and the development of the fluid-film thickness in the compliant contacts are still very rare.

3 ANALYSIS AND CONCLUSION OF LITERATURE REVIEW

3.1 Analysis of literature review

The presented research provides an overview of the issue of non-lubricated and lubricated compliant contacts. This is closely related to the operation of gears made of polymer materials in polymer/polymer or polymer/metal gearsets operating as fully compliant or hybrid, respectively [10]. The polymer materials used in the studies [8]-[10], [13],[111],[115],[118],[121] include PEEK, PA 6, PA 66, POM, PMMA, PDMS, PU, etc. In terms of mechanical properties, polymers generally exhibit a strong dependence on T compared to metals [16],[20], see Fig. 1. The service temperature interval of polymer machine elements is usually limited by the glass-transition temperature; when exceeded, the mechanical properties decrease sharply due to the rearrangement in the internal polymer structure.

The mechanical properties of the polymers (see Table 1) can be improved by modifiers such as glass and carbon fibers and other additives [7],[9]. On the other hand, the modification of the polymer by these fibers often leads to a deterioration of the tribological properties, such as CoF or wear rate [38]. Although some polymers, such as PC and PMMA [22], implemented in the experiments [8]-[10],[113],[114],[121], are not commonly used to design polymer gears, they are characterized by mechanical properties similar to those of engineering polymers [8],[38] and by the excellent light transmittance due to their amorphous structure, which allows for their use in optics [21]. Also, fillers, modifiers and additives are often an integral part of the polymer base material [20]. However, the polymer materials listed in Table 1 often have a semicrystalline or crystalline structure [1],[11], and thus exhibit low light transmittance, which limits the application of optical methods such as the optical interferometry [108].

Except for pitting, the overloading and melting wear belong to the most common causes of wear of the polymer gears [8]-[10]. Especially, under dry conditions, the increase in temperature is the result of friction in the contact, which significantly reduces the fatigue resistance due to the thermal degradation of the internal polymer structure [39],[40]. However, it has been proven that the geometry of the gear teeth and the pressure angle (α_G) of the teeth have a significant influence on the manifestation of the pitting wear [10],[50]. Load has a negative effect on both the lubricated and non-lubricated compliant contacts [8]-[10]. When the critical load is exceeded, a significant plastic deformation of the rubbing surfaces can occur, resulting in an intensive wear of polymer bodies due to overloading [43].

Under dry conditions, the rising temperature inside the contact causes an increase of the CoF, thereby an increase in wear rate, which has a direct impact on the service life of the polymer gear [8],[9]. This effect can be further enhanced by the use of different materials in gearsets (e.g., steel/polymer), where the extensive plastic deformations for the polymer gear were observed, due to a strong dependence of mechanical properties on the temperature [10]. The effect of the temperature has been studied mainly under the dry conditions [8],[43]-[40] and using internal solid lubricants [39].

For externally lubricated polymers [10],[114],[121], only a limited experimental evidence has been collected to analyze in detail the thermal effect that causes a decrease in the mechanical properties of the polymer. The film thickness between the rubbing surfaces of the polymer/polymer gearset is affected by the generally low thermal conductivity and diffusivity [21], which results in a higher temperature gradient in the contact region compared to the polymer/steel gearset, and consequently in a reduction of the film thickness [49]. Therefore, the solution may be coating of the polymer gears to prevent heat transfer into the substrate material similarly as for the coated steel gears. Thus, this may reduce the bulk temperature as well as the flash temperature in the contact of the polymer machine elements [8],[14],[42],[44].

The flash temperature in the contact area of the teeth increases due to the friction between the mating gears and, simultaneously, due to the overall heating of the gearset associated with the increase in bulk temperature [43]. Temperature has a significant effect on the mechanical and tribological properties as well as on the lubrication process [20],[39],[43]-[40]. An increase in CoF is manifested by a heat generation in the contact region, which results in a decrease in the film thickness of the lubricant and thus to a decrease in operating efficiency. Conversely, an increase in temperature leads to a decrease in the contact stiffness and thus, to an increase of fluid-film thickness according to the EHL theory.

In EHL theory, the stiffness of the contact is described by the E_R [64],[70]. In contrast to steels, the stiffness of polymers is significantly lower, which leads to a noticeable deformation of the rubbing surfaces of the viscoelastic contact bodies [73],[92]. With respect to the gears, the deformation of the rubbing surfaces leads to a change in the curvature of the involute curve of the tooth and, thus to an enlargement of the contact area between the mating gears [9].

Under ideal EHL conditions, the contact operates under fully flooded conditions, corresponding to $\Lambda \geq 3$ [53],[54], where the contact of surface asperities is prevented by a coherent fluid-film thickness. The main source of friction is attributed to the hydrodynamic friction in the liquid [55], represented by the η , which increases with the gradient of the U_E [52] (see Fig. 2). This effect is reflected in the parameters g_V and g_E , which define the individual modes of EHL in the hydrodynamic maps [67],[73] (see Fig. 3).

Steel gears are typically operated in the P-E mode of EHL, where non-conformal surfaces are in contact, resulting in high contact pressures [73],[95],[102]. In this regime, a significant dependence of the η on the p is characterized by the α [81]-[90]. Thus, high contact pressures ($p > 1$ GPa) are a consequence of the material properties of contact solids and the pressure-viscosity response of the lubricant causing the elastic deformation of the rubbing surfaces [10],[15],[70]. Therefore, this effect is recognized in hard EHL prediction formulae of fluid-film thickness by the $\bar{G}$ parameter [97]. For these rigid contacts, the viscosity response to pressure and temperature is measured by high-pressure viscometers and considered in rheological models, such as the relative volume model. This allows a quantitative description of the contact response to the lubricant flow in the EHL. For this reason, the reference lubricants with known properties are commonly used. Then, the accurate calculation of the fluid-film thickness from EHL models may be achieved.

Conversely, the externally lubricated polymer gears can operate in the I-E mode of EHL or in the TR region between the I-E and P-E modes. According to [67],[91],[95], the TR region is considered to be the operating region of polymers lubricated with mineral oils. In general, the contact pressures in the I-E mode are relatively low ($p < 1$ GPa), typically in the range from 1 to 100 MPa [92]-[95],[121].

As a result of the low E_R of the contact pair, the importance of the g_E is increased and the contact, according to the hydrodynamic map of EHL, operates in the I-E mode of EHL [73],[95],[96]. In the I-E mode, the η contributes significantly to the formation of the fluid-film thickness as was documented in [114],[121]. Despite the low contact pressures in the I-E mode, the contact pressure is sufficient to induce the elastic deformation of the contacting solids, but insufficient to initiate an increase in η . Therefore, the fluid-film thickness is not greatly affected, unlike the contacts operated in the P-E mode of EHL. In addition to the W and other parameters, the magnitude of the viscosity parameter g_V depends on the α [67],[73].

However, in the I-E mode, the influence of α is considered negligible, and thus $\bar{G}$ is omitted in the fluid-film thickness prediction formulae, while in the P-E mode, the influence of α is significant [70]. Due to the dependence of η on the p , a pressure peak, see Fig. 4, is formed at the exit of the contact [97],[114]. This effect is particularly noticeable at higher loads, where the pressure profile becomes similar to the Hertzian profile [56],[58]. Since the TR region is a transition region between the I-E and P-E modes of EHL, the influence of α can be expected, especially near the boundary between these modes. Unfortunately, the necessary experimental evidence for this is still missing [95],[96],[109].

With respect to the main operating parameters in the EHL, the fluid-film thickness is inversely proportional to the load and gradually decreases with increasing W [8],[114],[121]. In both P-E and I-E, the area of the contact is enlarged with increasing W , however, the h_m and h_c changes only slightly [121]. In the I-E mode, the effect of W is a more pronounced

than in the P-E mode due to the low E_R [73],[112]. This is reflected in the fluid-film thickness prediction models by the power exponent of the $\bar{W}$ [91],[93],[96].

In accordance with the above, H&D [60],[97] suggested a contribution of load W in the prediction formulae of h_c and h_m in P-E and I-E modes. For the former [60], the exponent of the $\bar{W}$ is -0.067, while for the latter [97], its value is -0.21. Dearn et al. [10] found that the fluid-film thickness in the steel/polymer contact is smaller than in the polymer/polymer contact, which is probably due to the magnitude of the E_R . The results of [115] can confirm these conclusions. However, the exponent of the $\bar{W}$ for the TR region has not yet been defined.

The increasing trend of the fluid-film thickness with the U_E has been experimentally documented in [10],[105],[117],[118],[121],[123] as a consequence of the increase in the shear stress between the lubricant-separated rubbing surfaces [81]. In the compliant contact at low U_E , the film thickness profiles demonstrated an almost uniform horizontal trend in the central region of the contact, but this was different for the inlet and exit regions [118]. At the exit of the contact, the constriction appears at low U_E [117]. However, this statement contradicts the results of [114],[121], where the constriction occurs at high U_E , similar to rigid contacts.

According to [100],[105], the h_m at low U_E and high W , the h_m is usually identified at the side lobes of the contact, while at high U_E , the h_m is shifted closer to the exit from the contact. However, according to [121], the h_m at different U_E was always observed at the side lobes of the horseshoe shape film. This is not consistent with Hooke's assumption of a shift of h_m from the exit to the side lobes with increasing U_E or decreasing W [93],[100].

Rolling speed and sliding speed are two main components of U_E [123]. As the sliding speed increases, the shear stresses in the lubricant increase and, thus, the film thickness decreases [115],[121]. This is referred to as the SRR, which characterizes the kinematic conditions in the lubricated contacts with respect to the physical contact of the mating gears. This results in the development of both components along the pressure line [10], with the exception of the pitch point on the pressure line, where only pure rolling should occur [10],[11]. When the sliding speed component predominates in SRR, the temperature increase occurs in the contact region as well as in the vicinity of the contact [12],[123]. In addition, it is essential to analyze the influence of the kinematic conditions with respect to the hysteresis of the viscoelastic polymers [47],[48].

During the relative motion of the rubbing surfaces, the effect of R_q on the film thickness is different for compliant materials [12],[107] than for rigid materials [71],[72]. Due to the low stiffness of polymers, loading of the contact region results in a smoothing of the surface asperities [45],[46]. As the R_q decreases, there is an increase in the h_m between the rubbing surfaces occurs [10]. Conversely, CoF increases with R_q , resulting in a decrease in h_m and a transition to the mixed lubrication regime [58].

The surface roughness of polymer contact solids limits the application of experimental optical methods for determining the film thickness, such as the LIF method [70],[114],[118],[119]. The effect of surface roughness negatively affects the quality of the interferograms obtained and makes it more difficult to evaluate the experimental data. In contrast, the method of optical interferometry [112],[114],[121] is more accurate than LIF and allows the evaluation of the film thickness in the order of nanometers [108]. The necessary conditions for the use of the optical methods are the light transmittance of one of the contact bodies and the reflectivity of the other.

The low surface roughness of the contacting bodies is a fundamental requirement for the operation of the contact in the EHL regime. In addition, the R_q limits the initialization of parasitic light reflection according to the asperity profile [116]. These requirements typically limit the selection of specimen materials to optical glass (BK-7) and sapphire (Al_2O_3) for hard EHL [80],[106], and PC and PMMA for the soft EHL [121],[122]. These optically smooth surfaces provide the best conditions for obtaining high quality interferograms, e.g., by the optical interferometry method.

3.2 Main conclusions of literature review

The review describes the issue from two main perspectives. The first one is the application of compliant materials in polymer gears operating under both dry and lubricated conditions. The second one is the simulation of the lubrication processes in the compliant contact of the polymer gears using a tribologically simplified contact geometry (not considering the complex tooth geometry). The former provides a coherent practical knowledge of the polymer materials, contact mechanics of polymer solids, viscoelastic behavior, failure modes of polymer gears, CoF, wear, thermal effects, and lubrication. The latter enables to identify the influence of individual operating parameters on the fluid-film thickness formation. In addition, the solved problem provides a necessary link of knowledge between the real-operating and the simulated-experimental conditions. The result of subchapters 3.1 and 3.2 is the analysis, interpretation and evaluation of the knowledge, which indirectly leads to the definition of the main aim of this dissertation thesis.

In the past, the issue of the soft EHL has been largely overlooked in contrast to the main research to the hard EHL. So far, only a few studies [10],[94],[107],[111],[114]-[119],[121] analyzing the formation of fluid-film thickness in soft EHL using an experimental approach have been published. Conversely, most of the research has focused on numerical models to predict fluid-film thickness and contact pressure for different contact geometries [91]-[100],[103]-[105],[111]. However, these models have not yet been sufficiently validated experimentally.

Formulae for H_c and H_m of the selected EHL models [91],[100],[114],[121] are summarized in Table 2 and Table 3, respectively, which quantitatively identify the influence of individual operating parameters especially in the soft EHL. The state of the art in the field of soft EHL is limited and further research is needed with respect to the rheology of liquids and the constitutive viscoelastic response of compliant solids. However, there is still a lack of knowledge that clarifies the formation of the fluid-film thickness in compliant contacts considering the effect of different kinematic conditions, load, temperature and SRR.

Table 2. Prediction formulae of dimensionless central film thickness in I-E and P-E modes of EHL for elliptical/circular contacts. a) [97], b) [91], c) [93], [100], d) [114], e) [121].

Author	Variable	Formula	Ref.
A)	H_c	$2.69 \cdot (1 - 0.61 \cdot e^{-0.73 \cdot k}) \cdot \bar{U}^{0.67} \cdot \bar{W}^{-0.067} \cdot \bar{G}^{0.53}$	[97]
B)		$7.32 \cdot (1 - 0.72 \cdot e^{-0.28 \cdot k}) \cdot \bar{U}^{0.64} \cdot \bar{W}^{-0.22}$	[91]
C)		$3.56 \bar{U}^{0.6} \cdot \bar{W}^{-0.13}$	[93],[100]
D)		$3.30 \bar{U}^{0.63} \cdot \bar{W}^{-0.13}$	[114]
E)		$2.60 \bar{U}^{0.55} \cdot \bar{W}^{-0.07}$	[121]

Table 3. Prediction formulae of dimensionless minimum film thickness at the contact exit h_{mr} and on side lobes h_{ms} in I-E and P-E modes of EHL for elliptical/circular contacts a) h_{mr} [97], b) h_{mr} [91], c) h_{mr} [93], [100], d) h_{ms} [93],[100], e) h_{mr} [114], f) h_{mr} [121].

Author	Variable	Formula	Ref.
A)	H_{mr}	$3.63 \cdot (1 - e^{-0.68 \cdot k}) \cdot \bar{U}^{0.68} \cdot \bar{W}^{-0.073} \cdot \bar{G}^{0.49}$	[97]
B)		$7.43 \cdot (1 - 0.85 \cdot e^{-0.31 \cdot k}) \cdot \bar{U}^{0.65} \cdot \bar{W}^{-0.21}$	[91]
C)		$2.83 \bar{U}^{0.6} \bar{W}^{-0.13}$	[93],[100]
D)	H_{ms}	$8.55 \bar{U}^{0.75} \bar{W}^{-0.23}$	[93],[100]
E)		$2.80 \cdot \bar{U}^{0.68} \cdot \bar{W}^{-0.20}$	[114]
F)		$2.8 \cdot \bar{U}^{0.62} \cdot \bar{W}^{-0.16}$	[121]

A major difference from the hard EHL is a significantly reduced stiffness of the contact bodies. In addition, polymers exhibit a strong dependence of their mechanical properties on the temperature, resulting in a constitutive viscoelastic response of the material. With respect to the effect of temperature, a several questions arise as to how the mechanical properties of polymers will be affected, how this will affect the lubrication process in terms of the expected combination of elastic, viscoelastic and even plastic deformations and how this will affect the formation of fluid-film thickness in compliant contact. Another important parameter is the surface roughness of the rubbing surfaces which, together with the implementation of the optical methods, influences the lubrication regime where the contact operates.

One of the fundamental shortcomings of the experimental studies is the frequent use of inappropriate materials of the contact pair, which do not represent the properties of high-performance and engineering polymers to design gears, although, the operating conditions of the contact correspond well to the soft EHL. Another shortcoming stems from the design of the experimental conditions that were often very far from the conditions under which polymer gears commonly operate. In addition, the soft EHL prediction models typically do not consider the contribution of the viscoelastic behavior of the polymers and the rheological response of the lubricant.

So far, the tribology of the compliant contacts has been studied mainly under dry and grease lubrication conditions as opposed to the compliant contacts lubricated with mineral and synthetic liquid lubricants. Therefore, the performed research is relevant and crucial in order to prevent the negative effects during the lubrication process of the polymer machine elements as well as to optimize the lubrication processes. However, the experimental evidence that would confirm or refute the findings of the soft EHL theory is still missing.

Some circumstances such as the inelastic response of the polymer or the rheological behavior of fluids, demonstrate the multidisciplinary nature of the topic. This topic has become of fundamental importance in connection with the efforts for the widespread use of the polymer machine elements in the mechanical engineering. Nowadays, engineering polymers are characterized by excellent mechanical properties that allow us to use them in high-performance applications where they have the potential to replace metals in the near future.

In soft EHL, the TR region between the P-E and I-E modes is considered to be an operating mode of the polymer gears lubricated with liquid lubricants. However, the knowledge of the formation of the fluid-film thickness in the TR region is very limited. This represents the so-called white space in the field of EHL theory.

4 RESEARCH AIM OF THESIS

The research aim of the present dissertation thesis is to clarify the tribological behavior of the compliant contacts operating in the EHL regime. The main attention is paid to the formation of the fluid-film thickness in the TR region between the I-E and P-E modes where the operation of the polymer gears is assumed. For this purpose, the film thickness will be investigated experimentally by the optical interferometry method with an emphasis to the effect of kinematic conditions, load, and temperature, considering viscoelastic response of the material and the rheological response of the lubricant. The research conducted will provide a description of the lubrication processes in the compliant contacts operated in the EHL regime. The observed results can contribute to the optimization of the operating conditions and thus to the increase of the efficiency and performance of the products using the polymer machine elements together with the prevention of the failure modes of such elements. The solution of the research aim includes the following research objectives:

- Analysis of the material properties of the selected polymer as a function of loading frequency and operating temperature,
- Analysis of the rheological properties of the selected lubricants as a function of pressure and operating temperature,
- Clarification of the influence of kinematic conditions and load on the formation of the fluid-film thickness in compliant contact,
- Clarification of the influence of operating temperature on the fluid-film thickness in compliant contact below and above the glass-transition temperature of the polymer,
- Comparison of experimental film thickness data with EHL models and determination of the validity of the I-E and P-E formulae for the TR region,
- Implementation of the constitutive viscoelastic response of the polymer material in the EHL models.

4.1 Scientific questions

General question Q:

- Will a coherent lubricant film always be formed in compliant contacts, fully separating the rubbing surfaces?

Specific questions Q1-Q3 of Q:

- Q1.** What is the effect of the load and entrainment speed on the fluid-film thickness and the contact shape in the compliant contact operated in the TR region of EHL?

- Q2.** How the rheological response of the lubricant contributes to the formation of the fluid-film thickness in the TR region of the EHL, considering a different effect in the I-E and P-E modes of the EHL?
- Q3.** What is the contribution of the constitutive viscoelastic response of the material in the compliant contacts to the formation of fluid-film thickness and contact shape changes in the TR region of EHL?

4.2 Hypotheses

H1. –

- a. *“The effect of load will be significantly dependent on the operating temperature below T_g , where the load-dependent behavior of the film thickness will be manifested.”*
- b. *“Near and above T_g , the load-dependent behavior of the fluid-film thickness will be negligible although a significant increase in the contact area and asymmetric deformation of the contact shape will be exhibited.”*
- c. *“The increase of the entrainment speed will cause a transition of minimum film thickness from the side lobes to the exit of the contact. However, the shape of the compliant contact will be only slightly affected.”*

H2. –

- a. *“In accordance with the EHL theory, the pressure-viscosity response of the lubricant to the formation of the fluid-film thickness is usually neglected in the I-E mode of EHL. However, this effect will be more significant in the TR region than in the I-E mode.”*

H3. –

- a. *“The increase in temperature will cause a significant decrease in material stiffness, and, consequently, in terms of the EHL theory, a shift from the TR region to the I-E mode of EHL, which will take effect by increasing the fluid-film thickness in the compliant contact.”*
- b. *“The temperature effect will be more critical for the conformational viscoelastic changes in the internal polymer structure influencing the contact shape rather than for possible defects by forming an unstable lubricant film and even above the T_g of solid material.”*

- c. *“An increase in the loading frequency of the solid material, referred to as the entrainment speed in the EHL, will make the compliant contact much stiffer, resulting in a decrease in the fluid-film thickness, although this effect will weaken with increasing temperature.”*

4.3 Thesis layout

In order to achieve the main aim of the dissertation thesis and to answer scientific questions, experimental measurements were carried out to investigate tribological processes in compliant lubricated contacts. According to the OECD (Organization for Economic Co-operation and Development) classification [130], the research focus corresponds well with the definition of the basic oriented research. The research performed was achieved in the field of science No. 2. *Engineering and technology*, in particular in the subfield No. 2.3 *Mechanical engineering* where the issue of tribology is included [130]. In addition, the research focus partially overlaps with the field of science No. 1. *Natural Sciences*, especially with Subfield 1.4. *Chemical Sciences*, which includes the *Polymer Science*.

This research was funded by the Czech Science Foundation (GACR), according to the discipline of Technical Sciences, Panel Specialization – P101 Mechanical Engineering [135]. The grant number of the project was 18-26849J. The research was carried out within the framework of Czech-German cooperation between the Brno University of Technology (BUT) and the Gear Research Center (FZG) of the Technical University Munich (TUM), and the Surface Engineering Institute (IOT) of the RWTH Aachen University.

Based on the research carried out, three research papers ([Paper A](#), [Paper B](#), [Paper C](#)) have been published only in the tribology and chemistry journals with the impact factor (Lubricants, Journal of tribology (ASME), and Polymers). In [Paper A](#), the effect of temperature and the entrainment speed on the lubrication process was investigated near and above the T_g of the polymer used. The contribution of the lubricant rheology to the formation of the fluid-film thickness in compliant contacts was studied in [Paper B](#) where, using low and high viscosity reference lubricants, and the influence of the load was determined. In [Paper C](#), the viscoelastic response of the material was determined by the nano-DMA and DMA analyses and subsequently implemented in the EHL prediction models. This paper covers the temperature interval below the T_g and thus touches especially the region of the linear viscoelasticity.

In addition, not yet mentioned [Paper D](#) is the paper in the conference proceedings. This paper deals with a variation of the fluid-film thickness due to the influence of the SRR. All papers ([Paper A](#), [Paper B](#), [Paper C](#), and [Paper D](#)) are indexed in the Web of Science (WoS) by DOI number.

Paper A. KRUPKA, Jiri, DOCKAL, Krystof, KRUPKA, Ivan and HARTL, Martin. Elastohydrodynamic Lubrication of Compliant Circular Contacts near Glass-Transition Temperature. Lubricants [online]. 13 July 2022. Vol. 10, no. 7, p. 155.



lubricants

- **Journal impact factor IF** =3.5
- **Quartile:** Q2, CiteScore = 4.1
- **DOI:** <https://doi.org/10.3390/lubricants10070155>
- **Author's contribution:** 75 %

Paper B. KRUPKA, Jiri, DOCKAL, Krystof, KRUPKA, Ivan and HARTL, Martin. Polymer Lubrication: Pressure–Viscosity–Temperature Dependence of Film Thickness for Highly Loaded Compliant Contacts in Elastohydrodynamic Lubrication Regime. Journal of Tribology [online]. 1 February 2023. Vol. 145, no. 2.



ASME DIGITAL COLLECTION
THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS

- **Journal impact factor IF**=2.5
- **Quartile:** Q3, CiteScore = 4.2
- **DOI:** <https://doi.org/10.1115/1.4055558>
- **Author's contribution:** 75 %

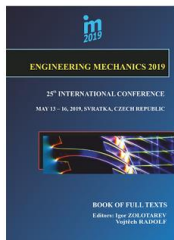
Paper C. KRUPKA, Jiri, DOCKAL, Krystof, SEDLACEK, Tomas, REBENDA, David, KRUPKA, Ivan and HARTL, Martin. Viscoelastic Response of Elastohydrodynamically Lubricated Compliant Contacts below Glass-Transition Temperature. Polymers [online]. 30 May 2023. Vol. 15, no. 11, p. 2528.



polymers

- **Journal impact factor IF**=5.0
- **Quartile:** Q1, CiteScore = 6.6
- **DOI:** <https://doi.org/10.3390/polym15112528>
- **Author's contribution:** 62 %

Paper D. KŘUPKA, J., ŠPERKA, P., KŘUPKA, I. and HARTL, M. LUBRICATION OF ROLLING - SLIDING POLYMER CONTACTS. In : Engineering Mechanics 2019 [online]. 13 May 2019. p. 219–222.



- Paper in the Conference Proceedings
- **DOI:** <https://doi.org/10.21495/71-0-219>
- **Author's contribution:** 75 %

5 MATERIALS AND METHODS

The dissertation deals with a multidisciplinary problem that, apart from of the tribology, partially overlaps with the material physics and chemistry. Laboratory experiments with regard to the basic oriented research ensure the accurate setting of steady-state operating conditions. At the same time, laboratory conditions provide a certain simplification in the simulation of the contact geometry, environmental conditions, etc. However, this allows us to separate the individual effects influencing the lubrication process.

With regard to the experimental nature of the dissertation thesis, all tribological measurements were carried out in the tribological laboratories of the Institute of Machines and Industrial Design (IMID) at Faculty of Mechanical Engineering, Brno University of Technology. The main attention has been paid to the influence of operating parameters such as U_E , T , or W , and to the analysis of the constitutive viscoelastic response of the material and the rheological response of the lubricant on the formation of the fluid-film thickness in compliant contacts. These experiments represent the essential majority of the research performed. Experiments focused on the development of the CoF represent only a minor part of the research.

In both cases, the experiments were performed on a rotational optical tribometer (Fig. 30a, hereafter referred to as “tribometer”), where the contact of the machine elements was simulated by a ball-on-disc configuration. In order to measure and evaluate the fluid-film thickness in the compliant contacts, the method of the optical chromatic interferometry [108] was implemented. In addition, the 3D optical profilometer (Fig. 30b, Bruker) was used to analyze the surface topography of the contact bodies before and after the tribological experiments.

The dynamic viscosity of the lubricants was determined on the basis of measurements on the RotoVisco1® (Fig. 30c, HAAKE) rotary viscometer and the high-pressure viscometer HPV (Fig. 30d, designed by Ing. Vojtech Polnicky in close cooperation with a respected tribologist and rheologist prof. Scott Bair, Ph.D., USA). However, the operating pressure interval of the viscometers used significantly exceeds the expected contact pressure interval for compliant contacts in the soft EHL. The viscoelastic response of the material was analyzed by nanoindentation (nano-DMA) using Hysitron TI Premier Nanoindenter (Fig. 30e, Bruker) and the dynamic mechanical analysis (DMA) using DMA1 (Fig. 30f, Mettler Toledo). A chemical analysis was performed by Raman spectroscopy using Renishaw inVia Reflex Raman spectroscope and Differential Scanning Colorimetry (DSC) by TA Instruments.



Fig. 30. Experimental equipment used. a) rotary optical tribometer (Brno University of Technology, Czech Republic), b) Bruker 3D optical profilometer (Bruker, USA), c) HAAKE RotoVisco1® (Thermo Fisher Scientific, USA), d) HPV (Brno University of Technology, Czech Republic), e) Hysitron TI Premier Nanoindenter (Bruker, USA), f) DMA1 (Mettler Toledo, Switzerland).

5.1 Selection of specimens

A compliant contact is formed between the rubbing surfaces of the contact bodies, usually in two configurations, as a “fully” compliant contact set (polymer/polymer), and also as a “hybrid” contact set (polymer/metal, composite, or glass). The latter configuration has been favored for the experiments. One of the main limitations in the selection of the specimen materials, is the internal structure of polymers, which can make the use of optical methods, such as the optical interferometry, very difficult.

The engineering and high-performance polymers commonly used in the gear design, such as PA 66, POM or PEEK, have a semi-crystalline internal structure arrangement associated with a low light transmittance, which limits the use of optical methods. On the other hand, both arrangements have a positive effect on the mechanical characteristics of the polymers. In addition to the light transmittance, this implies another requirement for the desired material, which should have mechanical properties similar to those of the above-mentioned polymers.

Both requirements are met by the amorphous polymers polycarbonate (PC) and polymethyl methacrylate (PMMA), which are appropriate for optical applications due to their excellent light transmittance, similar to the reference optical glass (BK-7) or sapphire (Al_2O_3) [116]. This is due to their amorphous arrangement of the internal structure and a low light absorption. Finally, the amorphous polymer PMMA was chosen for its mechanical characteristics, availability and higher light transmittance than the PC exhibits, see Table 4. The values given are general and may vary from manufacturer to manufacturer.

Table 4. Comparison of material properties of different amorphous materials at ambient room temperature

Property	PMMA	PC	BK-7 (ref.)	Sapphire
Elastic modulus, E (GPa)	3.3	2.3	76.0	345
Poisson's ratio, ν (-)	0.39	0.32	0.21	0.24
Coefficient of thermal expansion, ($\times 10^{-6}/^\circ\text{C}$)	68	68	7.1	6.66
Thermal conductivity, λ ($\text{W}/\text{m}\cdot^\circ\text{C}$)	0.2	0.2	11.1	23.1
Density, ρ (g/cm^3)	1.18	1.20	2.51	3.98
Operating temperature, T_w ($^\circ\text{C}$)	~90	~120	>400	>1600
Glass-transition temperature, T_g ($^\circ\text{C}$)	~105	~147	–	–
Refractive index, n (-)	1.49	1.58	1.52	1.77
Light transmittance, ϕ (%)	92	87	91	>90

The XT PMMA sheets (1x0.8x0.12 m) were purchased from a local commercial retailer in the Czech Republic before the start of the doctoral studies of the author of this thesis. The manufacturer of the PMMA sheets is NUDEC, S.A. (ES) [136]. A comparison of the material properties of PMMA with the reference optical glass BK-7 and sapphire, which are commonly used to simulate hard EHL contacts, is given in Table 4. For the selected materials, the R_q is very low, and hence the surface topography can be considered as optically smooth. Circular PMMA discs of 150mm and 120mm outer diameter and ~16mm inner diameter were then cut using a laser cutting machine. Finally, the PMMA specimens were coated (processed by an external company) by a thin chromium and antireflexive layer vapor-deposited on the top side and bottom side, respectively. The bottom side of the PMMA disc is in contact with a counterpart forming a compliant contact. The same pair of coatings is commonly applied, e.g., to BK-7 discs for rigid contacts in hard EHL when the optical interferometry method is used [108].

The counterpart to the PMMA disc is a bearing steel ball (100Cr6) with an outer diameter of 25.4 mm (1"). This steel is an equivalent to DIN 1.3505, ČSN 14109 and AISI 52100. The material properties of 100Cr6 steel are described in Table 1 and [54],[55]. The selected configuration (soft disc / hard ball) corresponds to the circular contact shape according to the Hertzian theory.

The second configuration (soft ball / hard disc) to form the compliant contact is to use a polymer ball in the contact pair with a glass BK-7 disc or a sapphire disc. The semi-crystalline polymers PA 66, POM and PEEK were selected for this purpose, see Fig. 31. All specimens were supplied with an outer diameter of 25.4 mm (1") by RGPBALLS S.r.l. ([137], Italy) for PA 66 and PEEK, and by REDHILL PRECISION BALLS Inc. ([138], USA) for POM.

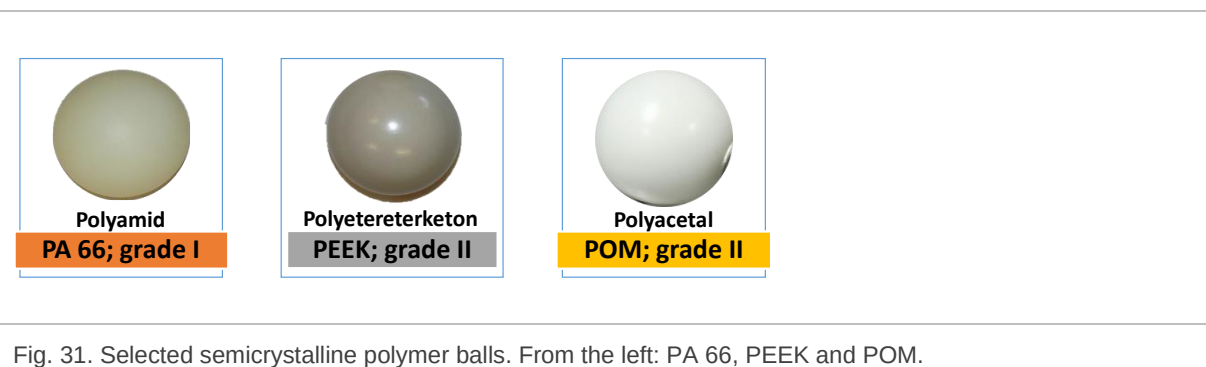


Fig. 31. Selected semicrystalline polymer balls. From the left: PA 66, PEEK and POM.

The material properties of selected balls are given in [Table 5](#). The dimensional and geometrical accuracy of polymer balls is determined by the “Grade” property, see [Table 6](#), which includes ball diameter tolerance, sphericity tolerance and finishing. Nevertheless, the certain limitation could be the surface roughness of polymer balls in contrast to the amorphous PMMA disc and a lower reflectivity of the surface. However, these characteristics are essential to ensure the operating conditions of the contact in the EHL regime and the use of optical methods.

Table 5. Material properties of selected semi-crystalline polymer balls.

Polymer property	PA 66	PEEK	POM
Elastic modulus, E (GPa)	2.50	3.75	2.80
Coefficient of thermal expansion, β ($\times 10^{-6}$ /°C)	87.5	55	93
Thermal conductivity, λ (W/m·°C)	0.25	0.28	0.27
Density, ρ (g/cm ³)	1.11	1.29	1.40
Service temperature, T_s (°C)	-30/80	-50/250	-40/110
Glass-transition temperature, T_g (°C)	~70	~143	~ -30

Table 6. Precision grades of selected semi-crystalline polymer balls.

Polymer property	PA 66	PEEK	POM
Grade	I	II	II
Tolerance on diameter, (μm)	± 25	± 50	± 50
Tolerance on sphericity, (μm)	Max ≤ 12	Max ≤ 25	Max ≤ 25
Surface finish operation	Lapped finishing	Grinded finishing	Grinded finishing

5.2 Selection of lubricants

In the accordance with the basic oriented research of this thesis, the reference synthetic, mineral, and natural liquid lubricants were selected for several criteria, see Fig. 32. Simultaneously, these reference lubricants are used worldwide in the basic and applied research not only in the field of EHL lubrication where, e.g., for FVA3, the term of “reference” lubricant is associated with its long-standing use within the Research Association for Drive Technology (Forschungsvereinigung Antriebstechnik) in Germany. The fundamental properties of the lubricant are the dynamic viscosity η and the pressure-viscosity coefficient α . The η indicates the resistance to internal friction in the fluid and is generally dependent on temperature, pressure and shear stress which cover the rheology of fluids.

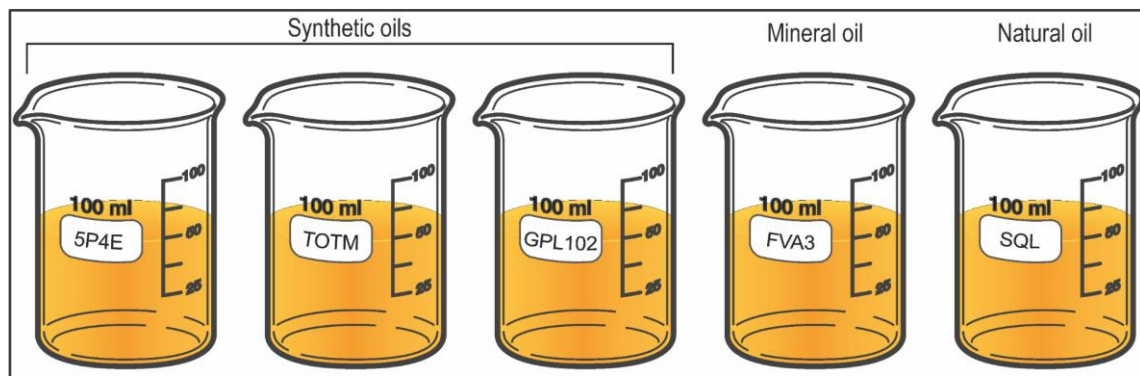


Fig. 32. The selected reference lubricants used in experiments.

The reference lubricants in Fig. 32 cover a wide viscosity interval of low-viscosity and high-viscosity lubricants represented by Squalane – SQL and Santovac 5 – 5P4E, respectively. The former is natural model liquid SQL ($\geq 99\%$) supplied by Sigma-Aldrich (lot BCBN6760V) and widely used for medical, cosmetic, and pharmacological purposes. The latter is Santovac[®] 5 – Polyphenyl Ether, also known as 5P4E or PPEs, a synthetic diffusion pump oil from SANTOLUBES LLC which belongs to the API (American Petroleum Institute) Group V according to base oil categories including synthetic lubricants such as polyglycols, silicones, esters, etc. One of the main characteristics of 5P4E lubricant is extreme viscosity, approximately $\eta_0 \approx 3 \text{ Pa s}$ at 24°C . Other exceptional lubricant properties also include resistance to chemical and thermal degradation up to 450°C ($\approx 180 \text{ K}$). Hence, it is usually used in aviation and aerospace applications involving extreme environmental and operating conditions.

The next lubricant is tri(2-ethylhexyl) trimellitate – TOTM supplied by Sigma-Aldrich (lot MKCF3309). TOTM is frequently used as a plasticizer of PVC and as the reference liquid (model fluid) in the field of rheology and the EHL. This liquid is classified in the API Group V including base fluids and additives. TOTM is a high viscosity lubricant that provides lubrication in high temperature applications covering engine and compressor oils.

Before the last in Fig. 32, Krytox® GPL 102 and GPL 105 – Perfluoropolyether (PFPE) are high performance synthetic reference lubricants that cover a wide range of industrial applications due to their universality. The main characteristics of these lubricants are superior thermal, chemical, and oxygen resistance, operation under extreme conditions of high temperatures and heavy loads, non-toxicity, non-flammability, non-reactivity with all common elastomers and plastics, etc. These lubricants were supplied by Sigma-Aldrich.

In addition to synthetic and natural lubricants, the FVA3 reference mineral oil with a viscosity index of ISO VG 100 (SAE 30) was used. The viscosity index of FVA3 corresponds to lubricants commonly used, for example in heavily loaded transmissions, air compressors, and hydraulic systems, etc. FVA3 was donated to us in cooperation with our colleagues from the Gear Research Center (FZG) of the Technical University of Munich (TUM) in Germany.

The selected lubricants are commonly used in experiments in rheology, EHL and also in practical applications. However, in accordance with the literature review, the scheduled experiments with the selected lubricants are novel and unique in the field of the soft EHL. The properties of the selected lubricants are given in Table 7.

Table 7. Selected properties of reference lubricants (40 °C, p = 0.1 MPa).

Lubricant	Atmospheric dynamic viscosity η_0 , Pa s	Pressure- viscosity coefficient α_{film} , Gpa ⁻¹	Refractive index n	Density ρ , kg·m ⁻³ at 15 °C
SQL	0.0157 at 40 °C [89]	18.1 at 40 °C [89]	1.44	810
GPL102	0.0285 at 40 °C [88]	33.8 at 40 °C [88]	-	-
FVA3	0.0840	20.2	1.481 at 25 °C	864
TOTM	0.0857 at 40 °C [88]	19.7 at 40 °C [88]	1.485 at 25 °C	915
5P4E	0.3700 at 40 °C	37.0 at 40 °C	1.630 at 25 °C	1198 at 25 °C

5.3 Preparation of tribological experiments

Once the materials of the contact bodies and lubricants have been selected, the two possible configurations for simulating the compliant contact have been defined in relation to the experimental method used. The first configuration is a contact between the semi-transparent PMMA disc (compliant body) and the steel ball (rigid body), see Fig. 33a. The second one is an inverted contact configuration, where contact is made between the semi-transparent BK-7 glass or sapphire disc and the PA66 or PEEK or POM balls, see Fig. 33b.

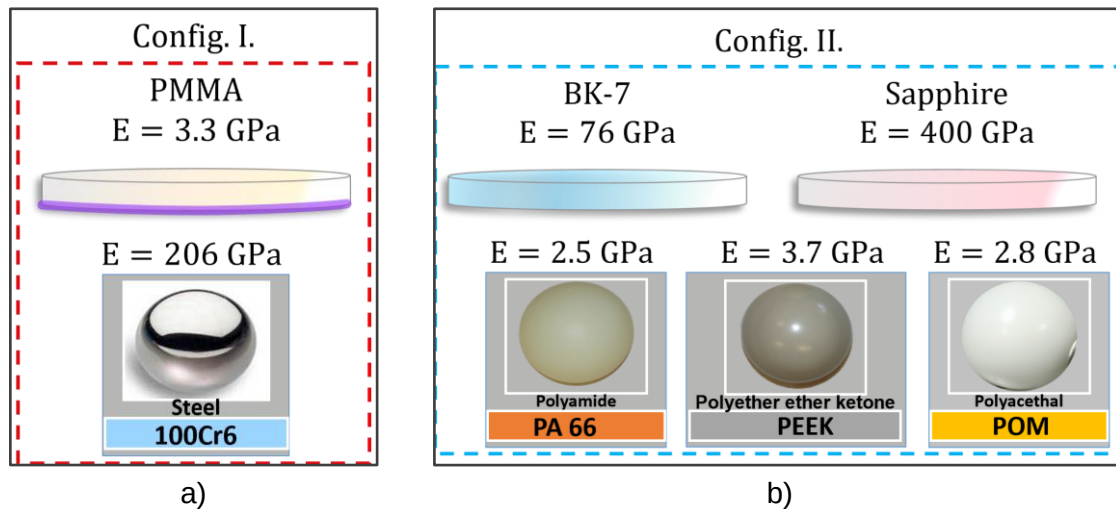


Fig. 33. Simulation of the compliant contact. a) soft disc / hard ball, b) hard disc / soft ball.

One of the preparation steps is an analysis of the surface topography of the specimens. In the present study, the surface topography is described by the experimentally obtained roughness parameters R_a and R_q . The surface roughness according to the ISO 25178-2 standard was measured with the commercial optical 3D profilometer ContourGT-X developed by Bruker. This profilometer employs a white light interferometry technique with a vertical resolution of 0.1 nm, and a repeatability of about 4.5 nm. The lateral resolution is defined by the 20x objective used and the 0.55x forward objective; for the values in Table 8, it is 0.86 nm. The raw data were analyzed according to the ISO 3274 standard with respect to the areal roughness definitions and the rules of the ISO 25178 standard.

Table 8. Surface roughness analysis of ball specimens - initial state (before polishing).

Polymer	Grade of ball	R_a , (μm)	R_q , (μm)
PA 66	I	1.61 ^(a) , 1.70 ^(b)	1.96 ^(a) , 2.01 ^(b)
PEEK	II	0.89 ^(a) , 0.36 ^(b)	1.15 ^(a) , 0.37 ^(b)
POM	II	0.38 ^(a) , X ^(c)	0.51 ^(a) , X ^(c)
100Cr6	-	0.010	0.012

Note (a) Measurements (profilometer), (b) RGPBALLS [137], (c) REDHILL PRECISION BALLS [138] (the producer does not state R_a or R_q).

According to Table 6, the ball grade characterizes the specimens in terms of their geometric and dimensional accuracy. Based on the results of the surface topography analysis, the initial values of R_a and R_q were determined and compared with the data provided by manufacturers [137] and [138], see Table 8. However, the surface roughness data, e.g., for PEEK, showed significant deviations from the values given by the manufacturers. A comparison of the initial surface topography between the 100Cr6 and POM balls is shown in Fig. 34a and Fig. 34b, which show a difference of one order.

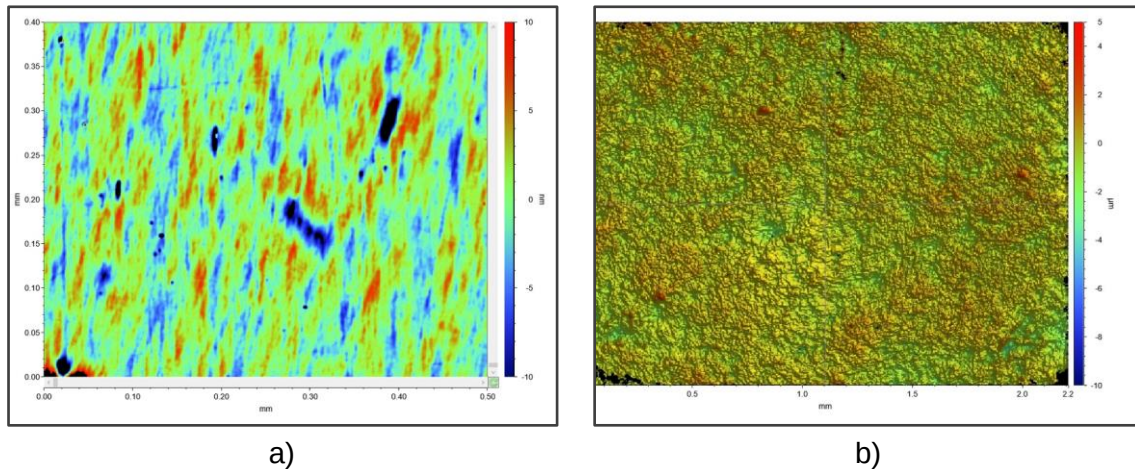


Fig. 34. Comparison of the initial topography of the steel and polymer ball specimens. a) 100Cr6, b) POM

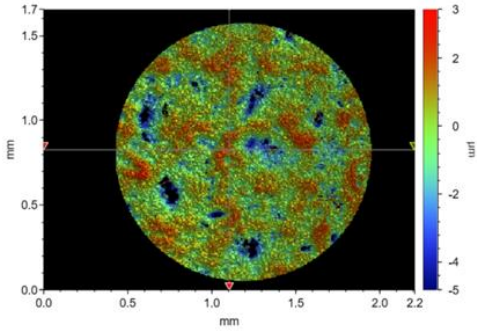
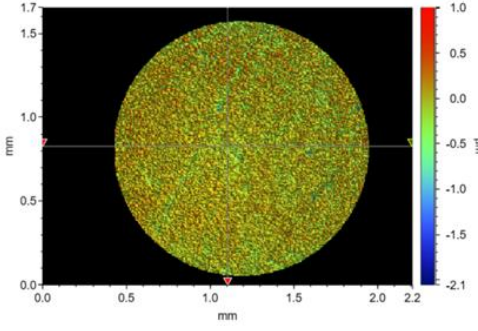
On the other hand, the surface of the counterparts of the balls, i.e., discs, can be considered as optically smooth, corresponding to $R_q < 5$ nm. However, for ball specimens, the tribological experiments are preceded by polishing of the contact surface in order to reduce R_q . After the polishing process, the surface roughness of the polymer balls was significantly reduced (approximately by 80 – 95 %) compared to the initial state. This is evident from comparing of both states in [Table 9](#) and

Table 10 for PA66 and PEEK, respectively. Unfortunately, the observed decrease in R_q is not sufficient to ensure the conditions of the EHL regime. Therefore, the second configuration (hard disc / soft ball) was excluded from the main part of the research concerned and the soft disc / hard ball configuration was preferred, which meets all the requirements to ensure the EHL regime.

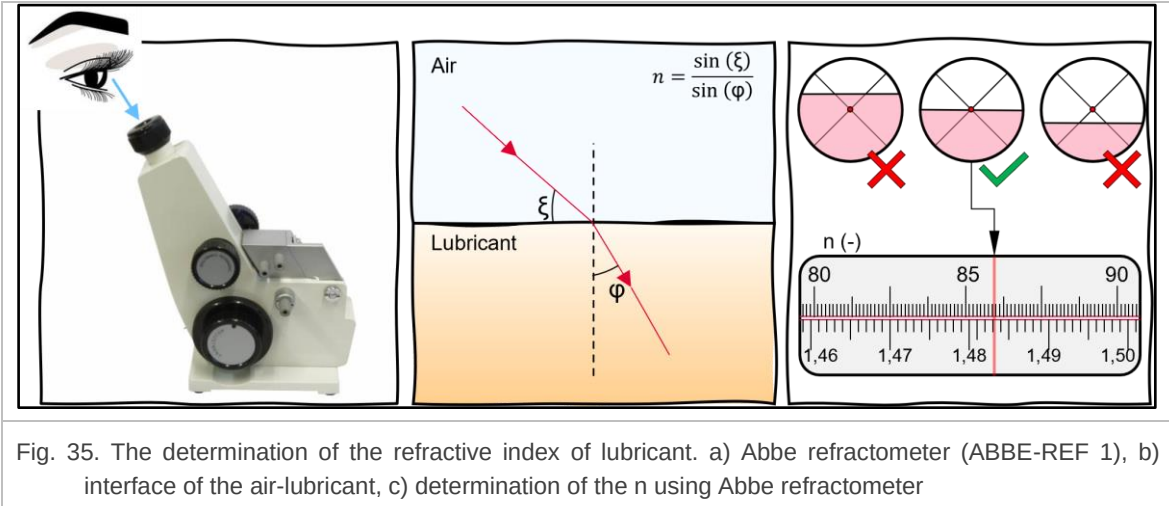
Table 9. Comparison of the topography of PA 66 balls before and after polishing process.

Polymer	Unpolished (new) ball	Polished ball
PA 66		
	$S_a = 1.70 \mu\text{m}$ $S_q = 2.05$ - Waviness + geom. Difference to ball $\approx 3.7 \mu\text{m}$	$S_a = 0.13 \mu\text{m} \rightarrow S_a$ decrease 92.5 % $S_q = 0.13 \rightarrow S_q$ decrease 91.6 % - Waviness + geom. Difference to ball $\approx 2.5 \mu\text{m} \rightarrow$ Geometry improved 32 %

Table 10. Comparison of the topography of PEEK balls before and after polishing process.

Polymer	Unpolished (new) ball	Polished ball
PEEK		
	• $S_a = 0.75 \mu\text{m}$ • $S_q = 1.00$ • Waviness + geom. Difference to ball $\approx 4.5 \mu\text{m}$	• $S_a = 0.14 \mu\text{m} \rightarrow S_a$ decrease 81.2 % • $S_q = 0.19 \rightarrow S_q$ decrease 80.9 % • Waviness + geom. Difference to ball $\approx 1.4 \mu\text{m} \rightarrow$ Geometry improved 69 %

The tribological experiments were always preceded by the determination of the refractive index (n) of the lubricants using the Abbe refractometer ABBE-REF 1 (PCE Instruments UK Ltd.), as shown in Fig. 35. Due to the characteristics of the apparatus used, the n in the present study was always determined at room temperature. This represents a certain simplification, while the dependence of n on temperature (when n decreases with T) was not considered.



5.4 Experimental apparatus

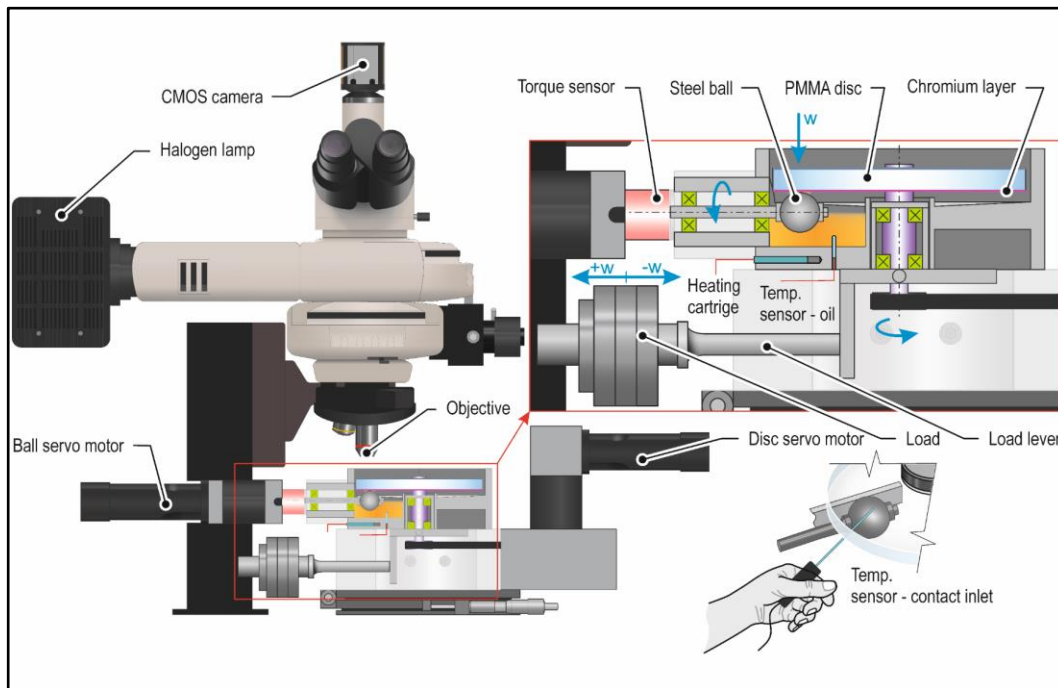


Fig. 36. Rotary optical tribometer in the ball-on-disc configuration.

The tribological experiments were performed on the optical rotary tribometer in a ball-on-disc configuration, see Fig. 36. The tribometer was used to model the tribological conditions in order to simulate the real operating conditions between the rubbing surfaces in accordance with the framework of the basic oriented research. Briefly, the rotational motion of both specimens is provided by a pair of independent servomotors controlled by the frequency converters, allowing different rotations and thus different SRRs to be set. At the same time, the axes of rotation of both specimens are perpendicular relative to each other.

The load is transmitted through the PMMA disc by means of a double lever when the PMMA disc is forced in the normal direction to the steel ball and a circular contact is formed. The magnitude of the load is determined by the weight of the steel cylinders and their position on the double lever, where the left limit position of the weight corresponds to the maximum W . In addition, the apparatus is equipped with a torque sensor, which is a crucial component, especially for CoF experiments.

The contact is illuminated using a halogen/LED lamp as the light source. The light passes further through the microscope to the Nikon CF Plan 5x/0.13 optical lens. High-speed cameras (CMOS and CCD cameras) from various manufacturers (PointGrey, Manta, etc.) were used to capture the contact region in high resolution and FPS.

The contact is tempered by several heating cartridges mounted near the oil reservoir and around the cylindrical body of the tribometer corresponding to the oil temperature and the bulk temperature, respectively. The tempering process is controlled by PLC unit that records and regulates the temperature. Furthermore, the temperature is monitored by a temperature sensor manually placed in the contact inlet (see Fig. 36).

5.5 Optical interferometry method

Optical interferometry is an optical method widely used in the precision engineering applications, such as 3D scanning of the surface topography or evaluation of fluid-film thickness in the tribology. In the field of EHL, the monochromatic and, later, chromatic interferometry have been used in the past. The advantage of the former is a wide measurable range of fluid-film thickness up to several microns. Unfortunately, the disadvantages include the lower precision and distinguishability than the latter. The framework developed by the tribology group of the Brno University of Technology refers to the optical colorimetric (chromatic) interferometry, which is described in more detail in [108].

In the microscope, which is part of the optical tribometer, there is a semipermeable mirror that reflects light from a light source, as shown in Fig. 37a. The light beam enters the contact where the semipermeable Cr coating reflects a part of the light. The Cr layer splits the light beam into two separate components.

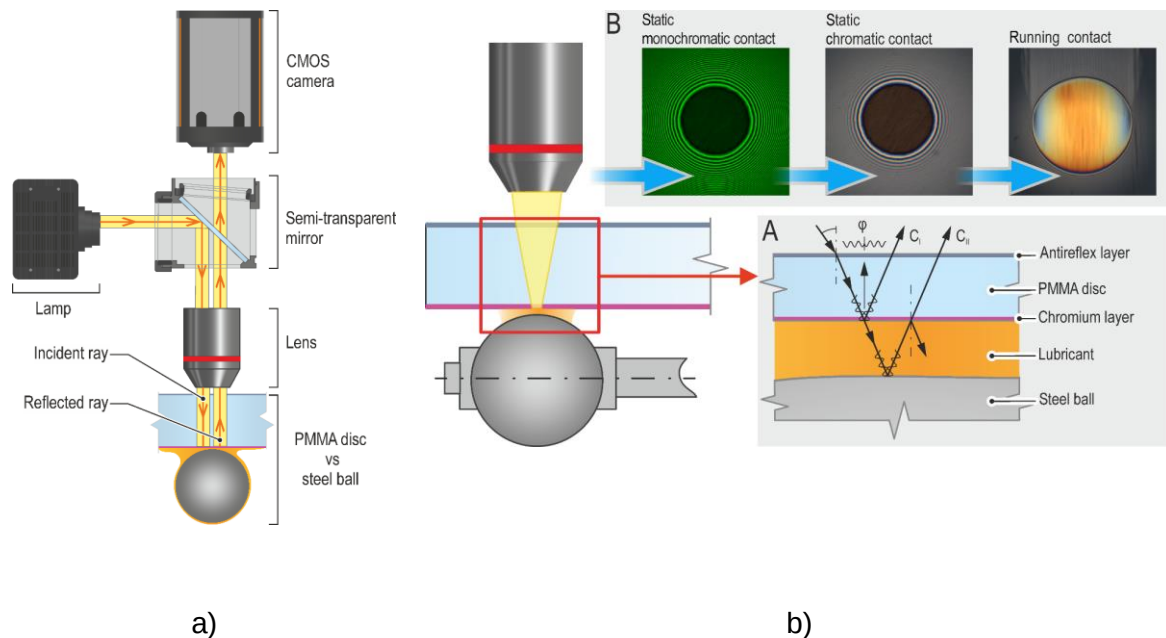


Fig. 37. Optical chromatic interferometry. a) sensing process, b) principle and calibration

The first part of the beam (C_1) is reflected from the Cr layer and returned to the microscope. The second part of the beam (C_2) passes through the Cr layer through the lubricant and is reflected from the surface of the steel ball. Each of these beams travels a different distance. Their phases (i_1 and i_2) are shifted with respect to each other. The difference in distance between them is calculated based on the interference of light wavelengths. These two beams cause interference due to the phase shift (ϕ).

The ϕ is equal to the distance that light travels from the surface of the disc to the ball and back. The individual waves carry the information about the magnitude and shape of the fluid-film thickness. In the absence of Cr coating, there is very little reflection of light from the bottom side of the disc, which make the implementation of the experimental method, difficult, see the detail A in [Fig. 37b](#). In addition, the anti-reflective coating prevents parasitic light emissions from penetrating the disc. Light interference is captured by the CCD cameras, and the interference patterns (interferograms) are processed in the unique software. The fluid-film thickness was measured, calibrated, and evaluated in Achilles software (ver. 4.0.117, Radek Poliscuk, Brno University of Technology, Brno, Czech Republic).

The process of evaluating the film thickness depends on the CIELAB color-film thickness calibration between monochromatic and chromatic interferograms of the static contact under steady-state conditions by comparing the light intensities assigned to a color spectrum representing the magnitude of the fluid-film thickness. The resolution of used method is smaller than one nanometer and the measurable range of film thickness is limited from 1 to 900 nm. A green filter ($\lambda = 564$ nm) was always used to obtain a monochromatic interferograms. A calibration process is illustrated in the detail B of [Fig. 37b](#).

5.6 Overview of experimental conditions

The experimental conditions were designed according to the aims of the dissertation thesis. Based on this, the specimen materials and lubricants were selected, and the preparation steps prior to the tribological experiments were carried out. The experimental apparatus and the optical chromatic interferometry method were introduced and described.

According to the preparation steps, the central and minimum film thicknesses were calculated based on the soft and hard EHL models (see [Table 2](#) and [Table 3](#)) and the results were used to design the experimental conditions. Finally, the parameters, such as entrainment speed, load, or temperature, were summarized in [Table 11](#), which provides a general overview of the experimental conditions. The experimental conditions corresponding to each research paper ([Paper A](#), [Paper B](#), [Paper C](#), and [Paper D](#)) are briefly described in the following chapter.

Table 11. General overview of the experimental conditions of the research conducted.

Property	Interval / Value
Entrainment speed, U_E (m/s)	10^{-4} – 10^1
Normal load, W (N)	5 – 100
Temperature, T (°C)	22 – 130
Dynamic viscosity at 40 °C, η (Pa s)	0.0157 – 0.37
Pressure-viscosity coefficient at 40 °C, α_{film} (GPa ⁻¹)	18.1 – 37.0
Sliding-rolling ratio, SRR	-1 – 1
Ellipticity, k	1

6 RESULTS AND DISCUSSION

The following text summarizes the results achieved and provides a discussion over the scientific problem solved. The findings of the individual research papers ([Paper A](#), [Paper B](#), [Paper C](#)) are shortly commented; the full text can be found in the selected papers [131]-[133]. The conference proceedings paper ([Paper D](#)) is also included [134].

In [Paper A](#), a compliant contact was operated near the glass-transition region of the PMMA to investigate the effect of the temperature on the formation of the fluid-film thickness and the contact shape. A compliant contact was simulated between the semi-transparent PMMA disc and a steel ball corresponding to the geometry where $k = 1$. The mineral lubricant FVA3 was used, and the contact was operated under pure rolling conditions ($SRR = 0$). The operating conditions were designed to include the temperature effect on the formation of the fluid-film thickness near and above the T_g interval (widely reported average value $\sim 105^\circ\text{C}$ [25] for the PMMA), where the dominant viscoelastic response of compliant contact bodies is assumed. Thus, the experiments cover the T interval from 90°C to 110°C . In addition, the contact was operated under two different loads (W , 20 and 50 N).

The interferograms of the static contacts ($U_E = 0$) were post-processed, and the contact radius R_c was evaluated. Then, according to the Hertzian theory, the contact radius a was substituted by R_c , which allows to reversibly derive the E_R and, consequently, E_1 for PMMA. The E_1 data were compared with the DMA experimental data [33], which were in good qualitative agreement, but differed quantitatively by about 18%. The obtained values of E_1 were then used to calculate h_c and h_m from of the soft EHL models. Further, the ellipticity k and the contact area A were evaluated. For the static contacts, the k corresponds well to the circular contact, where $k \in \langle 1.01, 1.06 \rangle$. However, A increased differently depending on T and W , where above T_g , an asymmetric deformation of the originally circular contact shape was documented.

The effect of temperature was found to be more significant for changes in the contact shape rather than for defects in the development of h_c . In addition, the increase of h_c with temperature and the load above T_g was manifested. This may indicate a minor influence of η and a major influence of the constitutive material behavior on the formation of the film thickness under given conditions. However, h_c was almost independent of W below T_g , where only a slight difference in h_c was observed for the loads selected. On the other hand, above T_g , the importance of W was greatly increased. This may also point out to exceeding the yield strength of PMMA. Then, the material exhibits strain softening or hardening depending on temperature, pressure (load), and strain rate.

Compared to the soft EHL models, the results showed a good agreement with the H&D model [91] above T_g , where the difference is less than 5%. However, below T_g , the h_c does not agree with this model and differs even more with the increase of U_E . The h_m was always identified in a horseshoe shape at the side lobes, which is characteristic of the hard EHL.

At $U_E \neq 0$, the contact shape demonstrated a change due to the influence of T , W , and U_E . Subsequently, the k increases steadily with T , resulting in a shortening of the contact in the direction parallel to U_E and its extending in the direction perpendicular to U_E . The results showed that U_E contributed to the increase of k only at 110 °C, while, at lower temperatures, the k was almost constant. In the former case, an increase in U_E causes a gradual deformation from the initially circular ($k = 1$) to the wide elliptical contact ($k > 1$, see Figure 7 in Paper A).

The subject of the interest in Paper B is to determine the dependence of the pressure-viscosity-temperature response of the lubricant on the formation of the fluid-film thickness in a compliant contact operated close to the TR region between the I-E and P-E modes of the EHL. The variation of η with hydrodynamic pressure and consequently with film thickness is particularly significant in the P-E mode of EHL, where it is associated with high stiffness of the contacting solid materials, such as metals or ceramics. In contrast, the polymers exhibit a significant compliance characteristic for the operation of the contact at low and very low contact pressures in the I-E mode of EHL.

In Paper B, the contact is simulated with the identical configuration used in Paper A and later in Paper C and Paper D. For simplicity, the contact was operated under pure rolling conditions ($SRR = 0$) to avoid initialization of the Couette component of the hydrodynamic friction. Two reference lubricants, SQL and TOTM, were used. The experiments were carried out for different loads (10 – 50 N) corresponding to p_H (Eq. (7)) up to 100 MPa at T of 24 °C and 40 °C, identical for the HPV experiments and the tribometer experiments. The dynamic viscosity function $\eta(p)$ of the reference fluids was obtained from the HPV experiments. The pressure-viscosity coefficients α_0 , α^* , and α_{film} were then calculated.

A comparison of HPV data with rheological models showed good agreement with the relative volume model [83],[87] as well as with the McEwen model [84], where only slight differences (up to 2 %) were observed. In addition, the HPV data were quite consistent with the results obtained by Bair [88]-[90]. In the second part of Paper B, the fluid-film thickness and the changes of the contact shape were investigated as a function of U_E , W , T , and η . The results demonstrated good agreement with the Hertzian theory for circular contact, i.e., R_c increased symmetrically in both perpendicular and parallel directions to the U_E in dependence on W from 24 °C to 40 °C. The E_1 of PMMA was then calculated according to Eq. (6). This showed only a slight deviation (up to 2.9 % on average) compared to Paper A.

Regarding the lubricant used, the h_c and h_m results differed significantly qualitatively and quantitatively. The SQL showed an almost linear increase in h_c as a function of U_E ; however, the TOTM showed a non-linear increase in h_c for both inlet temperatures. However, the interval of U_E differs for each lubricant in terms of the film thickness that can be measured by the experimental method used, being 0.2 – 1.0 m/s for SQL and 0.025 – 0.4 m/s for TOTM. This might be one of the possible sources of the discrepancy due to the different dynamic cyclic loading of the compliant contact and the hydrodynamic effect of the lubricant. On the other hand, both lubricants showed a similar behavior in terms of the increase of the deviation of h_c between individual loads with increasing U_E .

In addition, there was a noticeable deviation between h_{ms} and h_{mr} with U_E . However, a minimum was always identified at the side lobes of the horseshoe shape ($h_{ms} < h_{mr}$). Thus, the transition from the side lobes to the contact exit with increasing U_E or decreasing W proposed by Hooke [98]-[100] and later observed by Marx et al. [121] was not documented in Paper B. For TOTM, the h_m showed a marked load dependency, where its increase resulted in a significant difference between h_{ms} and h_{mr} . For SQL, significant fluctuations in h_{ms} were documented with U_E , probably due to the contribution of low viscosity that allows the lubricant to leak from the contact, resulting in unsteady fluid-film thickness development.

A comparison of the experimental data of h_c and h_m with the soft EHL prediction models revealed a significant discrepancy when the deviation, especially for TOTM, exceeds 30 %. A discrete comparison of the data showed a different trend with respect to the lubricant used; however, for TOTM, this deviation was always positive ($h_{c(exp)} > h_{c(mod)}$) regardless of the inlet temperature T , in contrast to SQL where a negative deviation was mostly documented ($h_{c(exp)} < h_{c(mod)}$). In addition, the experimental data of h_c and h_m were also qualitatively and quantitatively different from the hard EHL models. Thus, the question arises whether the contact operates closer to the I-E or P-E mode and how this is reflected in the prediction formulae of h_c and h_m .

The elastic modulus of PMMA is relatively high ($E \sim 3$ GPa) in the used temperature interval (up to 40 °C) compared to other polymers, e.g., PDMS, which shifts the operating region of the compliant contact more towards the P-E mode of the EHL. Similarly, with increasing η , the operating mode of EHL is closer to the P-E mode, where the pressure-viscosity effect of the lubricant is exhibited, as opposed to the I-E mode. According to the hydrodynamic map of EHL proposed by Johnson [73], the contact in Paper B was operated in the TR region between the I-E and P-E modes, where the influence of the lubricant rheology should not be neglected. However, the experimental results of this study qualitatively demonstrate a high sensitivity of the fluid-film thickness to the lubricant used and a low validity of the I-E and P-E EHL models for the TR region.

Documented deviations from the selected models result from the contribution of the individual model parameters ($\bar{U}$, $\bar{W}$, $\bar{G}$, and k). In this context, α_{film} proposed by Bair et al. [88]-[90] was implemented in the calculation of $\bar{G}$ in the H&D model [91]. Then, the new power exponents of $\bar{G}$ were obtained for both lubricants based on the regression of the h_c data.

However, it appears that the $\bar{G}$ is more relevant for TOTM than for low-viscosity SQL, where quite low values of the $\bar{G}$ exponent result from a non-linear regression. Thus, the exponent of $\bar{G}$ is significantly different for each lubricant; $\bar{G} \sim 0.084$ and $\bar{G} \sim 0.001$ for TOTM and SQL, respectively. These results in Paper B demonstrate a significant discrepancy in the effect of the lubricant rheology on the formation of the fluid-film thickness depending on the selected lubricant.

The Paper C deals with the viscoelastic response of elastohydrodynamically lubricated compliant contacts below T_g . This complements the results of Paper A which investigated the formation of the fluid-film thickness near the T_g interval. In this paper, the constitutive viscoelastic response of the material to the formation of the fluid-film thickness was considered. Therefore, the material analysis of PMMA polymer was engaged.

The storage E' and loss E'' moduli and the damping factor $\tan \delta$ of the PMMA polymer were determined using two experimental methods, the nanoindentation (nano-DMA) and dynamic mechanical analysis (DMA), by executing the frequency and temperature sweep measurements. This allowed to compose the master-curves of the above parameters by implementing the TTSP. Based on TTSP, the experimental data were regressed, the shift factors a_T were determined, and the constants C_1 and C_2 were derived according to the WLF equation (see Eq. (3)). As a result of the experiments performed, the frequency and temperature dependence of the selected polymer was determined, which corresponds to the operating conditions under dynamic cyclic loading of the contact during the tribological experiments. However, the data from the nano-DMA and DMA methods differ significantly relative to each other with respect to the frequency and temperature.

This discrepancy can be explained by a different scale of both methods and the inhomogeneous polymer structure across the material. The nano-DMA data correspond well to the surface structure of the solid, where the polymer solid has a different stiffness than the bulk structure of the solid. However, the DMA data are more representative of the bulk properties of the solid. In the second part of Paper C, the dependence of fluid-film thickness on T and U_E was determined under constant single W using the 5P4E lubricant. During the experiments, the contact was tempered on several (24, 40, 70, and 80 °C) discrete temperatures. The static contacts were evaluated in a similar manner as in Paper A and Paper B based on R_c . The dimensions of the contact in x-y coordinates were almost identical, corresponding to the circular contact shape ($k \sim 1$) and an almost linear increase with temperature.

Except for 24 °C, the static contact was always dimensionally smaller than the Hertzian contact radius a , where only the elastic response of the material is generally assumed. However, this may be somewhat inaccurate with respect to the expected constitutive viscoelastic response of PMMA. The contact area (A) showed a significant increase of about 30% in the temperature range from 24 to 80°C. This is different for the running contact ($U_E \neq 0$) which was always dimensionally smaller than the static contact and where the non-equivalence of R_c in x-y coordinates was observed, revealing only a slight transition from the circular to wide elliptical contact shape (where $k \in (1.03, 1.10)$).

Based on this, the E_1 was calculated via R_c and then integrated in I-E as well as P-E EHL models for comparison with experimental data. The difference of H_c experimental data from these models demonstrated a significant dependence on T . At 24 °C, predisposition to I-E or P-E models was not observed, and the measured data of H_c was underestimated or overestimated relative to these models, respectively. However, at 40 °C, the best agreement with I-E model was recorded, which quantitatively differs only about 2 % on the average. Qualitatively, the development of H_c also corresponds very well. Surprisingly, after the T exceeds 40 °C, the measured H_c correlates best with P-E model although the E_1 of polymer decreased with T and thus the contact should become more compliant. Moreover, the difference from the I-E model increases even more with the function of T . This points out to the discrepancy for which the origin seems not to be clear yet.

One explanation may be a different constitutive material response of polymers compared to the EHL models assumptions, where the Hookean (elastic) response of the material is expected. Also, the explanation may result from different experimental conditions (often described by $\bar{U}$ and $\bar{W}$) as opposed to the conditions for which the EHL models are derived and potentially valid. Not least explanation may be the operation of the contact in the different EHL mode relative to the hydrodynamic maps (often described by g_E and g_V).

Based on the hydrodynamic map, the contact was operated in the TR region between P-E and I-E modes. In addition, the predisposition to the former and the latter was recorded as a function of T , which affects both η of the lubricant and E of the polymer. Moreover, according to the Archard's parameter (g_4 versus H and g_E, g_V), the experimental conditions correspond well to the TR region. This may justify a difference between the measured H_c and the EHL models for P-E and I-E modes. The H_m was always identified at the side lobes in the horseshoe film shape, without transition to the contact exit, consistently with [Paper A](#) and [Paper B](#).

Using the generalized Maxwell model and Prony series, the relaxation modulus $E_r(t)$ was derived and the viscoelastic response of PMMA, based on the nano-DMA data, was integrated into the EHL models (subscript "m") as a time-dependent function. At 24 and 40 °C, only minor changes were observed, with H_c correlating best with hard and soft H&D for the former and latter temperatures, respectively. However, at 70 and 80 °C, the H_c showed very good agreement with soft H&D_m, with a deviation of only 10%, where previously the difference for soft H&D was more than 50%.

With respect to the hydrodynamic map, after using $E_r(t)$, the operational conditions were slightly shifted towards the P-E mode where the shift increment increases with T . In contrast, according to the g_4 (Figure. 14), $\hat{H}_c$, $\hat{H}_{mr}$, $\hat{H}_{ms}$ as well as g_E asymptotically approach the boundary between the TR region and I-E mode of EHL where $g_4 \approx 0.1$. The observed results conflict with the determination of the EHL mode on the one hand and the validity of the prediction formulas for the I-E and P-E modes of EHL in the TR region on the other hand. The observed results may be a consequence of the viscoelastic response of the polymer material resulting in a decrease of the fluid-film thickness, which is not accounted for in the EHL models.

The [Paper D](#) of the conference proceedings describes the formation of fluid-film thickness in the compliant contact operating under different SRR ratios. The SRR ratio is associated with the majority of engineering applications where always at least a slight difference occurs between the surface speeds of the rubbing surfaces of machine elements. This results in the initialization of heat in the contact and an increase in the CoF, which is described in detail by the rolling (Poiseuille flow) and sliding (Couette flow) friction components.

The development of h_m demonstrated an insignificant dependence on the listed SRRs. In addition, a narrowing of the horseshoe shape, especially at the side lobes, was observed with increasing SRR. The observed results are consistent with those obtained by Marx et al. [\[121\]](#), where a significant decrease in film thickness due to the effect of SRR was observed for the compliant contact only under pure sliding conditions.



Article

Elastohydrodynamic Lubrication of Compliant Circular Contacts near Glass-Transition Temperature

Jiri Krupka ^{*}, Krystof Dockal , Ivan Krupka and Martin Hartl

Faculty of Mechanical Engineering, Brno University of Technology, Technicka 2896/2,
602 00 Brno, Czech Republic; krystof.dockal@vut.cz (K.D.); krupka@fme.vutbr.cz (I.K.);
hartl@fme.vutbr.cz (M.H.)

^{*} Correspondence: jiri.krupka@vut.cz

Abstract: Lubrication of polymer materials nowadays represents a subject of interest in many engineering applications, such as bearings or gears, to utilize them in the areas where conventional metal materials have so far dominated. However, material properties of polymers are strongly dependent on temperature that delimits a lubrication process and leads to manifestations of viscoelastic behavior of polymers. An understanding of mechanisms, which are responsible for formation of film thickness near the glass-transition temperature, is necessary to prevent initialization of failure modes and to increase the durability of polymer engineering components. Optical chromatic interferometry was applied to investigate development of film thickness and changes in contact geometry of compliant circular contacts operated in the elastohydrodynamic lubrication regime (EHL). Film thickness was compared with soft EHL prediction models, differences in contact geometry were assessed and their contribution to film thickness development were evaluated. Qualitatively good agreement of experimental results of central film thickness and soft EHL predictions was observed; however, minimum film thickness shows significant discrepancies. Outcomes and findings confirm the operation of the compliant circular contact in Isoviscous-elastic regime of EHL and the main influence of temperature and load to thermomechanical response of amorphous polymer PMMA.



Citation: Krupka, J.; Dockal, K.; Krupka, I.; Hartl, M.

Keywords: elastohydrodynamic lubrication; Isoviscous-elastic regime; compliant circular contact; film thickness; glass-transition temperature; amorphous polymer; optical chromatic interferometry

Paper A:

Paper A:

Paper A:

Paper A:

Paper A:

Paper A:

Paper A:

Paper A:

Paper A:

Paper A:

Paper A:

Paper A:

Paper A:

Paper A:

Paper A:

Jiri Krupka¹

Department of Tribology,
Faculty of Mechanical Engineering,
Brno University of Technology,
Technická 2896/2,
616 69 Brno, Czech Republic
e-mail: jiri.krupka@vut.cz

Krystof Dockal

Department of Tribology,
Faculty of Mechanical Engineering,
Brno University of Technology,
Technická 2896/2,
616 69 Brno, Czech Republic
e-mail: krystof.dockal@vut.cz

Ivan Krupka

Department of Tribology,
Faculty of Mechanical Engineering,
Brno University of Technology,
Technická 2896/2,
616 69 Brno, Czech Republic
e-mail: krupka@fme.vutbr.cz

Martin Hartl

Department of Tribology,
Faculty of Mechanical Engineering,
Brno University of Technology,
Technická 2896/2,
616 69 Brno, Czech Republic
e-mail: hartl@fme.vutbr.cz

Polymer Lubrication: Pressure–Viscosity–Temperature Dependence of Film Thickness for Highly Loaded Compliant Contacts in Elastohydrodynamic Lubrication Regime

The relevance of the compliant contacts operated in elastohydrodynamic lubrication regime has increased during the last decades. Polymers and elastomers have been preferred because of their low-cost production or their tribological performance in many mechanical and bioengineering applications, where the metals originally dominated. Especially, in high-performance applications, such as polymer gears, the current subject of interest covers the transition between Piezoviscous-elastic and Isoviscous-elastic regimes of elastohydrodynamic lubrication. Here, it is necessary to precisely determine operating conditions and lubricant properties such as rheology whose contribution to film thickness formation may be influenced by attributes of individual lubrication regimes. The high-pressure viscosimeter and the optical tribometer were used, the former to establish the pressure–viscosity–temperature relationship of two reference lubricants, natural Squalane and synthetic tri(2-ethylhexyl) trimellitate, and the latter to determine the central and minimum film thickness in the circular contact between the PMMA disc and the steel ball using the optical chromatic interferometry method. Experimental results of film thickness demonstrated a significant deviation from the soft elastohydrodynamic lubrication (EHL) models, independently of the lubricant used, load, entrainment speed, and temperature because the pressure–viscosity–temperature response of lubricant was not included. Due to this, film thickness data were regressed, and new power coefficients of dimensionless parameter $\bar{G}$ were derived. Outcomes confirmed the operation of the compliant circular contact in the transition region between the Piezoviscous-elastic and Isoviscous-elastic regimes with the minimum film thickness identified on the side lobes of the horseshoe shape. [DOI: 10.1115/1.4055558]

Keywords: elastohydrodynamic lubrication, compliant contact, reference lubricants, lubricant rheology, film thickness, optical chromatic interferometry

Paper B:

Paper B:

Paper B:

Paper B:

Paper B:

Paper B:

Paper B:

Paper B:

Paper B:

Paper B:

Paper B:

Paper B:

Paper B:

Paper B:

Article

Viscoelastic Response of Elastohydrodynamically Lubricated Compliant Contacts below Glass-Transition Temperature

Jiri Krupka ^{1,*} , Krystof Dockal ², Tomas Sedlacek ³ , David Rebenda ^{1,4} , Ivan Krupka ¹ and Martin Hartl ¹

¹ Faculty of Mechanical Engineering, Brno University of Technology, Technická 2896/2, 616 69 Brno, Czech Republic; david.rebenda@vut.cz or rebenda@utb.cz (D.R.); krupka@fme.vutbr.cz (I.K.); hartl@fme.vutbr.cz (M.H.)

² HVM Plasma spol. s.r.o., Na Hutmance 2, 158 00 Praha 5, Czech Republic; krystof.dockal@hvm.cz

³ Centre of Polymer Systems, Tomas Bata University in Zlin, Trida Tomase Bati 5678, 760 01 Zlin, Czech Republic; sedlacek@utb.cz

⁴ Footwear Research Centre, University Institute, Tomas Bata University in Zlin, Nad Ovcimou IV 3685, 760 01 Zlin, Czech Republic

* Correspondence: jiri.krupka@vut.cz

Abstract: The widespread use of polymers in the high-performance engineering applications brings challenges in the field of liquid lubrication in order to separate the rubbing surfaces by the coherent fluid-film thickness relative to not only the inelastic material response of the polymers. The determination of the mechanical properties by the nanoindentation and the dynamic mechanical analysis represents the key methodology to identify the viscoelastic behavior with respect to the intense frequency and temperature dependence exhibited by polymers. The fluid-film thickness was examined by the optical chromatic interferometry on the rotational tribometer in the ball-on-disc configuration. Based on the experiments performed, first, the complex modulus and the damping factor for the PMMA polymer describing the frequency and temperature dependence were obtained. Afterwards, the central as well as minimum fluid-film thickness were investigated. The results revealed the operation of the compliant circular contact in the transition region very close to the boundary between the Piezoviscous-elastic and Isoviscous-elastic modes of the elastohydrodynamic lubrication regime, and a significant deviation of the fluid-film thickness from the prediction models for both modes in dependence on the inlet temperature.

Keywords: compliant contact; elastohydrodynamic lubrication; transition region; fluid-film thickness; optical chromatic interferometry; viscoelastic behavior



Citation: Krupka, J.; Dockal, K.; Sedlacek, T.; Rebenda, D.; Krupka, I.; Hartl, M. Viscoelastic Response of Elastohydrodynamically Lubricated Compliant Contacts below

Paper C:

Paper C:

Paper C:

Paper C:

Paper C:

Paper C:

Paper C:

Paper C:

Paper C:

Paper C:

Paper C:

Paper C:

Paper C:

Paper C:

Paper C:

Paper C:

Paper C:

Paper C:

Paper C:

Paper C:

7 CONCLUSIONS

In accordance with the basic oriented research, the present dissertation thesis deals with the formation of fluid-film thickness in compliant contacts operating in the EHL regime. The scientific problem solved is related to the multi-scale tribology, where the constitutive viscoelastic response of the polymer solids as well as the rheology of the liquids are reflected. This is the current topic with regard to the efforts for the widespread use of polymer machine elements in the high-performance engineering applications, where the dry conditions are unacceptable, and the grease lubrication is usually insufficient. Thus, it is necessary to operate the contact in the regime of liquid lubrication in order to reduce the friction between the rubbing surfaces to prevent the excessive wear that can lead to the failure of such elements.

Until recently, the research on the hard EHL has been more intensive than on the soft EHL, which represents one of the shortcomings resulting from the state of the art of compliant contacts. The studies published so far usually describe the operation in these regimes, but the knowledge about the formation of the fluid-film thickness in the TR region is very limited. In addition, the operating conditions, corresponding to the state of the art, are often quite different and do not consider the influence of the lubricant rheology and the phenomenon of viscoelastic behavior on the formation of the fluid-film thickness.

Based on the outcomes of the state of the art, the "white space" in the field of the EHL has been identified. In accordance with a critical analysis of the state of the art and the identification of "white space", the main aim of this dissertation thesis has been defined. Furthermore, the materials of the solid specimens and the lubricants for the research performed have been selected and the experimental methods and conditions have been introduced. With respect to the focus of the dissertation thesis, the individual research papers have been published in the selected journals with impact factor in the field of tribology and chemistry. Finally, a list of publications has been compiled.

The novel results of this dissertation thesis have been summarized in three impacted research papers indexed in the WoS and Scopus databases. The first paper was aimed at investigating the fluid-film thickness near the glass-transition temperature of the polymer. The main aim of the second paper was to investigate the pressure-viscosity effect of the lubricant on the formation of the fluid-film thickness in the compliant contact. The third paper focuses on the material analysis of the PMMA by nano-DMA and DMA and on the implementation of the constitutive viscoelastic response of the material below the glass-transition temperature into the EHL models.

The results of this dissertation thesis describe the formation of the fluid-film thickness in the compliant contact operating in the TR region between I-E and P-E modes. Thus, this thesis extends the knowledge in the field of the EHL of the compliant circular contacts by simulating the operating conditions, especially in relation to the lubrication of the polymer machine elements. However, the laboratory experiments presented use a simplified contact geometry with respect to the basic oriented research carried out. The main contribution of the thesis can be summarized as follows:

- Experimental evidence was obtained for the development of fluid-film thickness and contact shape changes in compliant contacts operated in TR region between the I-E and P-E modes of EHL,
- Implementation of the experimentally obtained viscoelastic response of the solid material and the rheological response of the lubricant into the EHL models,
- The formation of fluid-film thickness in compliant contacts near the glass-transition temperature of polymer was investigated, for the first time, by the optical interferometry method simulating the operating conditions that correspond well to the high-performance engineering applications.

The research conducted emphasizes the significance of the viscoelastic response of the material, which was experimentally obtained and implemented to the EHL models. However, this approach is still very rare in the soft EHL in contrast to the numerical modelling of the material response. Therefore, the attention paid to the viscoelastic response of the material is essential, and further experimental investigations of the lubrication processes in the compliant contacts are necessary.

In relation to the general scientific question (Q), the experimental evidence obtained can be summarized as follows:

- The lubricant film always fully separated the rubbing surfaces despite the contact shape changes.

In relation to the specific scientific questions(Q1-Q3) and hypotheses (H1-H3), the experimental evidence obtained can be summarized as follows:

- Below the glass-transition temperature of the polymer, the load-dependent behavior of the film thickness was observed, when the film thickness gradually decreased with magnitude of load. This effect became less significant with increasing inlet temperature. **(hypothesis H1.a was confirmed)**
- Near and above the glass-transition temperature of the polymer, the load-dependent behavior of the film thickness was not negligible. The film thickness demonstrated a dependence on the magnitude of the load without almost any change, or it even increased with the magnitude of the load. **(hypothesis H1.b was falsified)**

- The transition of the minimum film thickness with the entrainment speed from the side lobes to the exit of the contact was not recorded. The minimum was always observed at the side lobes of horseshoe shape. **(hypothesis H1.c was falsified)**
- The pressure-viscosity response of the lubricant should not be neglected in the TR region of the EHL where it contributes to the increase of the fluid-film thickness and to the shift of the operating conditions of the contact toward the boundary with the P-E mode of the EHL. **(hypothesis H2.a was confirmed)** The pressure-viscosity response may be further enhanced by the viscoelastic response of the contact solids.
- Below T_g , the film thickness gradually decreases with temperature as was expected, although above T_g , it was vice versa, resulting in an increase of film thickness. The increase in temperature resulted in a decrease in the contact stiffness and a shift to the I-E mode further away from the TR region according to the EHL map. **(hypothesis H3.a was confirmed)**
- The temperature effect was more critical for the conformational viscoelastic change in the internal polymer structure influencing the contact shape rather than forming of film thickness. The full separation of the rubbing surfaces was engaged, and defects forming an unstable lubricant film were not observed even above the T_g of solid material. **(hypothesis H3.b was confirmed)**
- Considering the viscoelastic response of the solid material by reduced elastic modulus, the contact becomes much stiffer and dimensionally smaller as was recorded with loading frequency (entrainment speed) causing the film thickness to decrease. This promotes the difference in film thickness from I-E models which overestimate the experimental film thickness, especially far away from the ambient temperature. **(hypothesis H3.c was confirmed)**

For the first time at our institute, the solved problem has been addressed as an original Ph.D. thesis. The results and findings of this thesis may be relevant and interesting for researchers in the field of soft EHL, polymer lubrication or engineering applications where polymers are widely used, or for anyone interested in tribological processes in compliant contacts. In all modesty, the author considers the research performed as a basic framework for future deeper investigations, especially considering the viscoelastic behavior of compliant materials. In addition to the hard disc/soft ball configuration, the viscoelastic response in the hard disc/soft ball configuration can also be investigated. Other options may include the use of different contact geometries or the inclusion of at least one other polymer material, which may be useful for future comparisons. The author considers the problem solved in the Ph.D. thesis to be relevant for the future research and would like to deal with it in more detail in the future.

LIST OF PUBLICATIONS

Papers published in journals with impact factor:

- KRUPKA, Jiri, DOCKAL, Krystof, KRUPKA, Ivan and HARTL, Martin. Elastohydrodynamic Lubrication of Compliant Circular Contacts near Glass-Transition Temperature. *Lubricants* [online]. 13 July 2022. Vol. 10, no. 7, p. 155. DOI: 10.3390/lubricants10070155
- KRUPKA, Jiri, DOCKAL, Krystof, KRUPKA, Ivan and HARTL, Martin. Polymer Lubrication: Pressure–Viscosity–Temperature Dependence of Film Thickness for Highly Loaded Compliant Contacts in Elastohydrodynamic Lubrication Regime. *Journal of Tribology* [online]. 1 February 2023. Vol. 145, no. 2. DOI: 10.1115/1.4055558
- KRUPKA, Jiri, DOCKAL, Krystof, SEDLACEK, Tomas, REBENDA, David, KRUPKA, Ivan and HARTL, Martin. Viscoelastic Response of Elastohydrodynamically Lubricated Compliant Contacts below Glass-Transition Temperature. *Polymers* [online]. 30 May 2023. Vol. 15, no. 11, p. 2528. DOI: 10.3390/polym15112528

Papers in conference proceedings:

- KŘUPKA, J., ŠPERKA, P., KŘUPKA, I. and HARTL, M. LUBRICATION OF ROLLING - SLIDING POLYMER CONTACTS. In : *Engineering Mechanics 2019* [online]. 13 May 2019. p. 219–222. DOI: 10.21495/71-0-219

REFERENCES

- [1] GILBERT, Marianne. Plastics Materials. In: GILBERT, Marianne (ed.), *Brydson's Plastics Materials* [online]. Eighth Edi. Elsevier, 2017. p. 1–18. ISBN 9780323358248.
- [2] BERER, M., HALB, M., FEUCHTER, M., PACHER, G. and PINTER, G. Fatigue Fracture Properties and Morphology of Polyoxymethylene (POM) Plates Produced under Moderate Processing Conditions. *International Journal of Polymer Science*. 2018. Vol. 2018, p. 38–40. DOI 10.1155/2018/7410925.
- [3] ISO 6336-1-6:2019 *Calculation of load capacity of spur and helical gears*. 3rd edition. Switzerland: International Organization for Standardization, 2019.
- [4] ANSI/AGMA 2001-D04-*Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*. USA: American Gear Manufacturers Association, 2004.
- [5] VDI 2736-4 *Thermoplastic gear wheels – Determination of strength parameters on gears*. Germany: The Association of German Engineers, 2016.
- [6] JIS B 1759 *Estimation of tooth bending strength of cylindrical plastic gears*. Japan: Japanese Standards Association, 2019.
- [7] SENTHILVELAN, S. and GNANAMOORTHY, R. Damage Mechanisms in Injection Molded Unreinforced, Glass and Carbon Reinforced Nylon 66 Spur Gears. *Applied Composite Materials* [online]. November 2004. Vol. 11, no. 6, p. 377–397. DOI: 10.1023/B:ACMA.0000045313.47841.4e.
- [8] HARRASS, M., FRIEDRICH, K. and ALMAJID, A. A. Tribological behavior of selected engineering polymers under rolling contact. *Tribology International* [online]. 2010. Vol. 43, no. 3, p. 635–646. DOI 10.1016/j.triboint.2009.10.003.
- [9] DEARN, K.D., HOSKINS, T.J., PETROV, D.G., REYNOLDS, S.C. and BANKS, R. Applications of dry film lubricants for polymer gears. *Wear* [online]. February 2013. Vol. 298–299, no. 1, p. 99–108. DOI: 10.1016/j.wear.2012.11.003.
- [10] DEARN, K. D., HOSKINS, T. J., ANDREI, L. and WALTON, D. Lubrication regimes in high-performance polymer spur gears. *Advances in Tribology*. 2013. Vol. 2013. DOI: 10.1155/2013/987251.
- [11] HOSKINS, T.J., DEARN, K.D., CHEN, Y.K. and KUKUREKA, S.N. The wear of PEEK in rolling-sliding contact – Simulation of polymer gear applications. *Wear* [online]. January 2014. Vol. 309, no. 1–2, p. 35–42. DOI: 10.1016/j.wear.2013.09.014.
- [12] LAUX, K. A., JEAN-FULCRAND, A., SUE, H. J., BREMNER, T. and WONG, J. S.S. The influence of surface properties on sliding contact temperature and friction for polyetheretherketone (PEEK). *Polymer* [online]. 2016. Vol. 103, p. 397–404. DOI: 10.1016/j.polymer.2016.09.064.
- [13] WALTON, D, TESSEMA, A A, HOOKE, C J and SHIPPEN, J M. Load Sharing in Metallic and Non-Metallic Gears. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* [online]. 1 March 1994. Vol. 208, no. 2, p. 81–87. DOI: 10.1243/PIME_PROC_1994_208_104_02.

- [14] HOLMBERG, K., RONKAINEN, H. and MATTHEWS, A. Tribology of thin coatings. *Ceramics International* [online]. August 2000. Vol. 26, no. 7, p. 787–795. DOI: 10.1016/S0272-8842(00)00015-8.
- [15] DOWSON, D. and HAMROCK, B. J. Numerical Evaluation of the Surface Deformation of Elastic Solids Subjected to a Hertzian Contact Stress. *A S L E Transactions* [online]. January 1976. Vol. 19, no. 4, p. 279–286. DOI: 10.1080/05698197608982804.
- [16] WINEMAN, A. Nonlinear Viscoelastic Solids—A Review. *Mathematics and Mechanics of Solids* [online]. 1 May 2009. Vol. 14, no. 3, p. 300–366. DOI: 10.1177/1081286509103660.
- [17] CHEN, Daniel T.N., WEN, Qi, JANMEY, Paul A., CROCKER, John C. and YODH, Arjun G. Rheology of Soft Materials. *Annual Review of Condensed Matter Physics* [online]. 10 August 2010. Vol. 1, no. 1, p. 301–322. DOI: 10.1146/annurev-conmatphys-070909-104120.
- [18] MEIDAV, TSVI. VISCOELASTIC PROPERTIES OF THE STANDARD LINEAR SOLID*. *Geophysical Prospecting* [online]. March 1964. Vol. 12, no. 1, p. 80–99. DOI: 10.1111/j.1365-2478.1964.tb01891.x.
- [19] CHRISTÖFL, Petra, CZIBULA, Caterina, BERER, Michael, ORESKI, Gernot, TEICHERT, Christian and PINTER, Gerald. Comprehensive investigation of the viscoelastic properties of PMMA by nanoindentation. *Polymer Testing* [online]. 2021. Vol. 93, no. xxxx, p. 106978. DOI: 10.1016/j.polymertesting.2020.106978.
- [20] FETTERS, L. J., LOHSE, D. J., RICHTER, D., WITTEN, T. A. and ZIRKEL, A. Connection between Polymer Molecular Weight, Density, Chain Dimensions, and Melt Viscoelastic Properties. *Macromolecules*. 1994. Vol. 27, no. 17, p. 4639–4647. DOI: 10.1021/ma00095a001.
- [21] DOS SANTOS, W. N., DE SOUSA, J. A. and GREGORIO, R. Thermal conductivity behaviour of polymers around glass transition and crystalline melting temperatures. *Polymer Testing* [online]. 2013. Vol. 32, no. 5, p. 987–994. DOI: 10.1016/j.polymertesting.2013.05.007.
- [22] CHAUDHRI, M.M. Impact breakage of semi-brittle spheres. *Powder Technology* [online]. June 2004. Vol. 143–144, p. 31–40. DOI: 10.1016/j.powtec.2004.04.006.
- [23] DJUKIC, Stéphanie, BOCAHUT, Anthony, BIKARD, Jérôme and LONG, Didier R. Mechanical properties of amorphous and semi-crystalline semi-aromatic polyamides. *Heliyon*. 2020. Vol. 6, no. 4. DOI: 10.1016/j.heliyon.2020.e03857.
- [24] DE DEUS, Jeferson F., SOUZA, Gabriel P., CORRADINI, Walimir A., ATVARS, Teresa D. Z. and AKCELRUD, Leni. Relaxations of Poly(methyl methacrylate) Probed by Covalently Attached Anthryl Groups. *Macromolecules* [online]. 1 September 2004. Vol. 37, no. 18, p. 6938–6944. DOI: 10.1021/ma049941c.
- [25] MATHIESEN, Danielle, VOGTMANN, Dana and DUPAIX, Rebecca B. Characterization and constitutive modeling of stress-relaxation behavior of Poly(methyl methacrylate) (PMMA) across the glass transition temperature. *Mechanics of Materials* [online]. 2014. Vol. 71, p. 74–84. DOI: 10.1016/j.mechmat.2014.01.003.
- [26] SANE, S. B. and KNAUSS, W. G. The time-dependent bulk response of Poly (Methyl Methacrylate). *Mechanics Time-Dependent Materials*. 2001. Vol. 5, no. 4, p. 293–324. DOI: 10.1023/A:1012517421899.

- [27] HIGGENBOTHAM-BERTOLUCCI, P. R., GAO, H. and HARMON, J. P. Creep and stress relaxation in methacrylate polymers: Two mechanisms of relaxation behavior across the glass transition region. *Polymer Engineering and Science*. 2001. Vol. 41, no. 5, p. 873–880. DOI: 10.1002/pen.10785.
- [28] WU, Souheng. Secondary relaxation, brittle–ductile transition temperature, and chain structure. *Journal of Applied Polymer Science*. 1992. Vol. 46, no. 4, p. 619–624. DOI: 10.1002/app.1992.070460408.
- [29] KLOMPEN, E. T.J. and GOVAERT, L. E. Nonlinear viscoelastic behaviour of thermorheologically complex materials: A modelling approach. *Mechanics of Time-Dependent Materials*. 1999. Vol. 3, no. 1, p. 49–69. DOI: 10.1023/A:1009853024441.
- [30] FEDERICO, C. E., BOUVARD, J. L., COMBEAUD, C. and BILLON, N. Large strain/time dependent mechanical behaviour of PMMAs of different chain architectures. Application of time-temperature superposition principle. *Polymer* [online]. 2018. Vol. 139, p. 177–187. DOI: 10.1016/j.polymer.2018.02.021.
- [31] MAZERAN, Pierre Emmanuel, BEYAOUI, Moez, BIGERELLE, Maxence and GUIGON, Michèle. Determination of mechanical properties by nanoindentation in the case of viscous materials. *International Journal of Materials Research*. 2012. Vol. 103, no. 6, p. 715–722. DOI: 10.3139/146.110687.
- [32] MENARD, Kevin P and MENARD, Noah. Dynamic Mechanical Analysis DYNAMICAL MECHANICAL ANALYSIS. *Encyclopedia of Analytical Chemistry* [online]. 2017. P. 1–22. DOI: 10.1002/9780470027318.a2007.pub3.
- [33] BOUMBIMBA, R. M., AHZI, S., BAHLOULI, N., RUCH, D. and GRACIO, J. Dynamic Mechanical Properties of PMMA/Organoclay Nanocomposite: Experiments and Modeling. *Journal of Engineering Materials and Technology* [online]. 1 July 2011. Vol. 133, no. 3, p. 1–6. DOI: 10.1115/1.4004052.
- [34] SCHIESSEL, H., METZLER, R., BLUMEN, A. and NONNENMACHER, T. F. Generalized viscoelastic models: Their fractional equations with solutions. *Journal of Physics A: General Physics*. 1995. Vol. 28, no. 23, p. 6567–6584. DOI: 10.1088/0305-4470/28/23/012.
- [35] COLBY, Ralph H. Breakdown of time-temperature superposition in miscible polymer blends. *Polymer*. 1989. Vol. 30, no. 7, p. 1275–1278. DOI: 10.1016/0032-3861(89)90048-7.
- [36] BARBERO, E. J. Time-temperature-age superposition principle for predicting long-term response of linear viscoelastic materials. *Creep and Fatigue in Polymer Matrix Composites*. 2010. P. 48–69. DOI: 10.1533/9780857090430.1.48.
- [37] BRINSON, L. C. and KNAUSS, W. G. Thermorheologically complex behavior of multi-phase viscoelastic materials. *Journal of the Mechanics and Physics of Solids*. 1991. Vol. 39, no. 7, p. 859–880. DOI: 10.1016/0022-5096(91)90009-D.
- [38] HOOKE, C. J., KUKUREKA, S. N., LIAO, P., RAO, M. and CHEN, Y. K. The friction and wear of polymers in non conformal contacts. *Wear*. 1996. Vol. 200, no. 1–2, p. 83–94. DOI: 10.1016/S0043-1648(96)07270-5.
- [39] KALIN, M. and KUPEC, A. The dominant effect of temperature on the fatigue behaviour of polymer gears. *Wear* [online]. April 2017. Vol. 376–377, p. 1339–1346. DOI: 10.1016/j.wear.2017.02.003.

- [40] MAO, K., LANGLOIS, P., HU, Z., ALHARBI, K., XU, X., MILSON, M., LI, W., HOOKE, C. J. and CHETWYND, D. The wear and thermal mechanical contact behaviour of machine cut polymer gears. *Wear*. 2015. Vol. 332–333, p. 822–826. DOI: 10.1016/j.wear.2015.01.084.
- [41] BLOK, H. The flash temperature concept. *Wear* [online]. 1 November 1963. Vol. 6, no. 6, p. 483–494. [Accessed 23 February 2018]. DOI: 10.1016/0043-1648(63)90283-7.
- [42] TOBE, Toshimi and KATO, Masana. A Study on Flash Temperatures on the Spur Gear Teeth. *Journal of Engineering for Industry* [online]. 1 February 1974. Vol. 96, no. 1, p. 78–84. DOI: 10.1115/1.3438333.
- [43] MAO, K. A new approach for polymer composite gear design. *Wear* [online]. February 2007. Vol. 262, no. 3–4, p. 432–441. DOI: 10.1016/j.wear.2006.06.005.
- [44] MAO, K. A numerical method for polymer composite gear flash temperature prediction. *Wear*. 2007. Vol. 262, no. 11–12, p. 1321–1329. DOI: 10.1016/j.wear.2007.01.008.
- [45] BIGGS, S. and SPINKS, G. Atomic force microscopy investigation of the adhesion between a single polymer sphere and a flat surface. *Journal of Adhesion Science and Technology* [online]. January 1998. Vol. 12, no. 5, p. 461–478. DOI: 10.1163/156856198X00164.
- [46] QUAGLINI, V. and DUBINI, P. Friction of Polymers Sliding on Smooth Surfaces. *Advances in Tribology* [online]. 2011. Vol. 2011, no. 1, p. 1–8. DOI: 10.1155/2011/178943.
- [47] MOYLE, N., WU, H., KHRIPIN, C., BREMOND, F., HUI, C.Y. and JAGOTA, A. Enhancement of elastohydrodynamic friction by elastic hysteresis in a periodic structure. *Soft Matter* [online]. 2020. Vol. 16, no. 6, p. 1627–1635. DOI: 10.1039/C9SM02087J.
- [48] SCARAGGI, M. and PERSSON, B.N.J. Theory of viscoelastic lubrication. *Tribology International* [online]. April 2014. Vol. 72, p. 118–130. DOI: 10.1016/j.triboint.2013.12.011.
- [49] SUKUMARAN, Jacob, ANDÓ, Mátyás, RODRIGUEZ, Vanessa, BAETS, Patrick DE and DANIEL NEIS, Patric. Friction torque, temperature and roughness in roll-slip phenomenon for polymer-steel contacts. . P. 7–16.
- [50] FERNANDES, P. J. L. and MCDULING, C. Surface contact fatigue failures in gears. *Engineering Failure Analysis* [online]. 1997. Vol. 4, no. 2, p. 99–107. DOI: [https://doi.org/10.1016/S1350-6307\(97\)00006-X](https://doi.org/10.1016/S1350-6307(97)00006-X).
- [51] MARTIN, H. Lubrication of gear teeth. *Engineering*. 1916, vol. 102, p. 199–204.
- [52] STACHOWIAK, G. W. a. W. BATCHELOR. *Engineering tribology*. Fourth edition. Amsterdam: Butterworth-Heinemann is an imprint of Elsevier, 2014. ISBN 978-012-3977-762.
- [53] HAMROCK, Bernard J., Steven R. SCHMID a Bo O. JACOBSON. *Fundamentals of Fluid Film Lubrication Book Fundamentals of Fluid Film Lubrication*. 2nd. Boca Raton: CRC Press, 2004. ISBN 9780429215315.
- [54] HANSEN, J., BJÖRLING, M. and LARSSON, R. A New Film Parameter for Rough Surface EHL Contacts with Anisotropic and Isotropic Structures. *Tribology Letters* [online]. 22 June 2021. Vol. 69, no. 2, p. 37. DOI: 10.1007/s11249-021-01411-3.

- [55] BJÖRLING, M., MIETTINEN, J., MARKLUND, P., LEHTOVAARA, A. and LARSSON, R. The correlation between gear contact friction and ball on disc friction measurements. *Tribology International* [online]. 2015. Vol. 83, p. 114–119. DOI: 10.1016/j.triboint.2014.11.007.
- [56] HERTZ. On the Contact of Elastic Solids. *Crelle's Journal*. Vol. 92, p. 156–171.
- [57] CAMERON, A. *Basic lubrication theory*. Ellis Horwood, 1981. Ellis Horwood series in engineering science. ISBN 0-85312-177-X.
- [58] MASJEDI, M. and KHONSARI, MM. Mixed lubrication of soft contacts: An engineering look. *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology* [online]. 5 February 2017. Vol. 231, no. 2, p. 263–273. DOI: 10.1177/1350650116652286.
- [59] REYNOLDS, Osborne. On the Theory of Lubrication and Its Application to Mr. Beauchamp Tower's Experiments, Including an Experimental Determination of the Viscosity of Olive Oil. *Philosophical Transactions of the Royal Society of London* [online]. 26 July 1886. Vol. 177, p. 157–234.
- [60] DOWSON, D, HIGGINSON, G R and WHITAKER, A V. Elasto-Hydrodynamic Lubrication: A Survey of Isothermal Solutions. *Journal of Mechanical Engineering Science* [online]. 6 June 1962. Vol. 4, no. 2, p. 121–126. DOI: 10.1243/JMES_JOUR_1962_004_018_02.
- [61] THEYSE, F.H. Some aspects of the influence of hydrodynamic film formation on the contact between rolling/sliding surfaces. *Wear* [online]. January 1966. Vol. 9, no. 1, p. 41–59. DOI: 10.1016/0043-1648(66)90013-5.
- [62] ROELANDS, C. J.A., VLUGTER, J. C. and WATERMAN, H. I. The viscosity-temperature-pressure relationship of lubricating oils and its correlation with chemical constitution. *Journal of Fluids Engineering, Transactions of the ASME*. 1963. Vol. 85, no. 4, p. 601–607. DOI: 10.1115/1.3656919.
- [63] ROELANDS, C. Correlational Aspects of the Viscosity-Temperature Pressure Relationship of Lubricating Oils. *PhD thesis*. 1966. No. April, p. 495.
- [64] DOWSON, D. Paper 10: Elastohydrodynamics. *Proceedings of the Institution of Mechanical Engineers, Conference Proceedings* [online]. 18 September 1967. Vol. 182, no. 1, p. 151–167. DOI: 10.1243/PIME_CONF_1967_182_014_02.
- [65] ARCHARD, J. F. Non-Dimensional Parameters in Isothermal Theories of Elastohydrodynamic Lubrication. *Journal of Mechanical Engineering Science* [online]. 6 April 1968. Vol. 10, no. 2, p. 165–167. DOI: 10.1243/JMES_JOUR_1968_010_023_02.
- [66] HERREBRUGH, K. Solving the Incompressible and Isothermal Problem in Elastohydrodynamic Lubrication Through an Integral Equation. *Journal of Lubrication Technology*. 1968. Vol. 90, p. 262.
- [67] GREENWOOD, J. A. An extension of the Grubin theory of elastohydrodynamic lubrication. *Journal of Physics D: Applied Physics* [online]. 1 December 1972. Vol. 5, no. 12, p. 309. DOI: 10.1088/0022-3727/5/12/309.
- [68] Moes, H.; and Bosma, R.: Film Thickness and Traction in EHL at Point Contact. Symposium on Elastohydrodynamic Lubrication, Inst. Mech. Engrs., 1972, p. 149.

- [69] HALL, R. W. and SAVAGE, M. D. Two-Dimensional Elastohydrodynamic Lubrication Part 2: Solution of the Line Contact Problem. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* [online]. 11 September 1988. Vol. 202, no. 5, p. 354–360. DOI: 10.1243/PIME_PROC_1988_202_133_02.
- [70] SPIKES, Hugh. Basics of EHL for practical application. *Lubrication Science* [online]. January 2015. Vol. 27, no. 1, p. 45–67. DOI: 10.1002/lis.1271.
- [71] ZHU, D and CHENG, H S. Effect of Surface Roughness on the Point Contact EHL. *Journal of Tribology* [online]. 1 January 1988. Vol. 110, no. 1, p. 32–37. DOI 10.1115/1.3261571.
- [72] MASJEDI, M. and KHONSARI, M.M. On the effect of surface roughness in point-contact EHL: Formulas for film thickness and asperity load. *Tribology International* [online]. February 2015. Vol. 82, no. PA, p. 228–244. DOI: 10.1016/j.triboint.2014.09.010.
- [73] JOHNSON, K. L. Regimes of Elastohydrodynamic Lubrication. *Journal of Mechanical Engineering Science* [online]. 6 February 1970. Vol. 12, no. 1, p. 9–16. DOI: 10.1243/JMES_JOUR_1970_012_004_02.
- [74] BAIR, Scott and KOTTKE, Peter. Pressure-viscosity relationships for elastohydrodynamics. *Tribology Transactions*. 2003. Vol. 46, no. 3, p. 289–295. DOI: 10.1080/10402000308982628.
- [75] BAIR, Scott. Pressure–viscosity response in the inlet zone for quantitative elastohydrodynamics. *Tribology International* [online]. May 2016. Vol. 97, no. January, p. 272–277. DOI: 10.1016/j.triboint.2016.01.037.
- [76] VERGNE, P. and BERTHE, D. Paper X(i) Pressure viscosity and compressibility of different mineral oils. In : *Tribology Series* [online]. 1987. p. 309–315.
- [77] EHRET, P., DOWSON, D. and TAYLOR, C. M. On lubricant transport conditions in elastohydrodynamic conjunctions. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*. 1998. Vol. 454, no. 1971, p. 763–787. DOI: 10.1098/rspa.1998.0185.
- [78] PONJAVIC, Aleks, CHENNAOUI, Mourad and WONG, Janet S.S. Through-thickness velocity profile measurements in an elastohydrodynamic contact. *Tribology Letters*. 2013. Vol. 50, no. 2, p. 261–277. DOI: 10.1007/s11249-013-0118-x.
- [79] SNOEIJER, J. H., EGGERS, J. and VENNER, C. H. Similarity theory of lubricated Hertzian contacts. *Physics of Fluids*. 2013. Vol. 25, no. 10, p. 1–7. DOI: 10.1063/1.4826981.
- [80] KRUPKA, I., KUMAR, P., BAIR, S., KHONSARI, M. M. and HARTL, M. The Effect of Load (Pressure) for Quantitative EHL Film Thickness. *Tribology Letters* [online]. 19 March 2010. Vol. 37, no. 3, p. 613–622. DOI: 10.1007/s11249-009-9559-7.
- [81] HÖGLUND, Erik. Influence of lubricant properties on elastohydrodynamic lubrication. *Wear* [online]. October 1999. Vol. 232, no. 2, p. 176–184. DOI: 10.1016/S0043-1648(99)00143-X.
- [82] BARUS, Carl. Isothermals, isopiestic and isometrics relative to viscosity. *American Journal of Science* [online]. 1 February 1893. Vol. s3-45, no. 266, p. 87–96. DOI 10.2475/ajs.s3-45.266.87.

- [83] MURNAGHAN, F. D. The Compressibility of Media under Extreme Pressures. *Proceedings of the National Academy of Sciences* [online]. 15 September 1944. Vol. 30, no. 9, p. 244–247. DOI: 10.1073/pnas.30.9.244.
- [84] McEwen, E., The Effect of Variation of Viscosity with Pressure on the Load- Carrying capacity of the Oil Film between Gear-Teeth, *J. Inst. Petroleum*, 1952, Vol.38, No.344-5, pp.646-672.
- [85] P.S.Y. Chu and A. Cameron, Pressure Viscosity Characteristics of Lubricating Oils, *Journal of the Institute of Petroleum*, 1962, Vol. 48, pp. 147-155.
- [86] JONES, William R., JOHNSON, Robert L., WINER, Ward O. and SANBORN, David M. Pressure-Viscosity Measurements for Several Lubricants to 5.5×10^8 Newtons Per Square Meter (8×10^4 PSI) and 149 C (300 F). *A S L E Transactions* [online]. 1 January 1975. Vol. 18, no. 4, p. 249–262. DOI: 10.1080/05698197508982767.
- [87] DOOLITTLE, Arthur K. Studies in Newtonian Flow. II. The Dependence of the Viscosity of Liquids on Free-Space. *Journal of Applied Physics* [online]. December 1951. Vol. 22, no. 12, p. 1471–1475. DOI: 10.1063/1.1699894.
- [88] BAIR, S. Reference liquids for quantitative elastohydrodynamics: selection and rheological characterization. *Tribology Letters* [online]. 1 May 2006. Vol. 22, no. 2, p. 197–206. DOI: 10.1007/s11249-006-9083-y.
- [89] BAIR, S. The temperature and pressure dependence of viscosity and volume for two reference liquids. *Lubrication Science* [online]. 1 March 2016. Vol. 28, no. 2, p. 81–95. DOI: 10.1002/ls.1302.
- [90] BAIR, S., LIU, Y. and WANG, Q. J. The Pressure-Viscosity Coefficient for Newtonian EHL Film Thickness With General Piezoviscous Response. *Journal of Tribology* [online]. 1 July 2006. Vol. 128, no. 3, p. 624–631. DOI: 10.1115/1.2197846.
- [91] HAMROCK, Bernard J. and DOWSON, Duncan. Elastohydrodynamic Lubrication of Elliptical Contacts for Materials of Low Elastic Modulus I—Fully Flooded Conjunction. *Journal of Lubrication Technology* [online]. 1 April 1978. Vol. 100, no. 2, p. 236–245. DOI: 10.1115/1.3453152.
- [92] BISWAS, S. and SNIDLE, R. W. Elastohydrodynamic Lubrication of Spherical Surfaces of Low Elastic Modulus. *Journal of Lubrication Technology* [online]. 1 October 1976. Vol. 98, no. 4, p. 524–529. DOI: 10.1115/1.3452912.
- [93] HOOKE, C. J. and O'DONOGHUE, J. P. Elastohydrodynamic Lubrication of Soft, Highly Deformed Contacts. *Journal of Mechanical Engineering Science* [online]. 22 February 1972. Vol. 14, no. 1, p. 34–48. DOI: 10.1243/JMES_JOUR_1972_014_008_02.
- [94] JAFFAR, M. J. Estimation of Minimum Thickness for Line Contacts in the Transition Region between the Isoviscous—Elastic and the Piezoviscous-Elastic Lubrication Regimes. *Proceedings of the Institution of Mechanical Engineers, Part C: Mechanical Engineering Science* [online]. 10 November 1989. Vol. 203, no. 6, p. 379–386. DOI: 10.1243/PIME_PROC_1989_203_131_02.
- [95] MYERS, T G, HALL, R W, SAVAGE, M D and GASKELL, P H. The Transition Region of Elastohydrodynamic Lubrication. *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences* [online]. 1991. Vol. 432, no. 1886, p. 467–479. DOI: 10.1098/rspa.1991.0026.

- [96] ESFAHANIAN, M. and HAMROCK, B. J. Fluid-Film Lubrication Regimes Revisited. *Tribology Transactions* [online]. January 1991. Vol. 34, no. 4, p. 628–632. DOI: 10.1080/10402009108982081.
- [97] HAMROCK, B J and DOWSON, D. Isothermal Elastohydrodynamic Lubrication of Point Contacts: Part III—Fully Flooded Results. *Journal of Lubrication Technology* [online]. 1 April 1977. Vol. 99, no. 2, p. 264–275.
- [98] HOOKE, C. J. The Elastohydrodynamic Lubrication of Heavily Loaded Contacts. *Journal of Mechanical Engineering Science* [online]. 22 August 1977. Vol. 19, no. 4, p. 149–156. DOI 10.1243/JMES_JOUR_1977_019_034_02.
- [99] HOOKE, C. J. Calculation of Clearances in Soft Point Contacts. *Journal of Tribology* [online]. 1 January 1988. Vol. 110, no. 1, p. 167–173. DOI: 10.1115/1.3261558.
- [100] HOOKE, C. J. The Elastohydrodynamic Lubrication of Elliptical Point Contacts Operating in the Isoviscous Region. *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology* [online]. 24 December 1995. Vol. 209, no. 4, p. 225–234. DOI: 10.1243/PIME_PROC_1995_209_433_02.
- [101] HAMROCK, B. J. and DOWSON, D. Film thickness for different regimes of fluid-film lubrication. [online]. 1983. No. May 1983.
- [102] BERGNER, L. and JENG, B. J. Piezoviscous effects in nonconformal contacts lubricated hydrodynamically. *ASLE Transactions*. 1986. Vol. 30, no. 4, p. 452–464. DOI: 10.1080/05698198708981779.
- [103] HOOKE, C. J. and HUANG, P. Elastohydrodynamic lubrication of soft viscoelastic materials in line contact. *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology* [online]. 1 March 1997. Vol. 211, no. 3, p. 185–194. DOI: 10.1243/1350650971542417.
- [104] HUNTER, S. C. The Rolling Contact of a Rigid Cylinder With a Viscoelastic Half Space. *Journal of Applied Mechanics* [online]. 1 December 1961. Vol. 28, no. 4, p. 611–617. DOI: 10.1115/1.3641792.
- [105] STUPKIEWICZ, Stanisław, LENGIEWICZ, Jakub, SADOWSKI, Przemysław and KUCHARSKI, Stanisław. Finite deformation effects in soft elastohydrodynamic lubrication problems. *Tribology International* [online]. January 2016. Vol. 93, p. 511–522. DOI: 10.1016/j.triboint.2015.03.016.
- [106] SPIKES, H. A. Thin films in elastohydrodynamic lubrication: The contribution of experiment. *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology* [online]. 1 May 1999. Vol. 213, no. 5, p. 335–352. DOI: 10.1243/1350650991542712.
- [107] KANETA, M, TODOROKI, H, NISHIKAWA, Hiroshi, KANZAKI, Y and KAWAHARA, Y. Tribology of Flexible Seals for Reciprocating Motion. *Journal of Tribology* [online]. 1 October 2000. Vol. 122, no. 4, p. 787–795. DOI: 10.1115/1.1310573.
- [108] HARTL, M., KRUPKA, I., POLISCUK, R., LISKA, M., MOLIMARD, J., QUERRY, M. and VERGNE, P. Thin Film Colorimetric Interferometry. *Tribology Transactions* [online]. January 2001. Vol. 44, no. 2, p. 270–276. DOI: 10.1080/10402000108982458.

- [109] ROBERTS, A D and TABOR, D. The extrusion of liquids between highly elastic solids. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences* [online]. 23 November 1971. Vol. 325, no. 1562, p. 323–345. DOI: 10.1098/rspa.1971.0172.
- [110] POLL, Gerhard and GABELLI, A. Formation of Lubricant Film in Rotary Sealing Contacts: Part II—A New Measuring Principle for Lubricant Film Thickness. *Journal of Tribology* [online]. 1 April 1992. Vol. 114, no. 2, p. 290–296. DOI: 10.1115/1.2920886.
- [111] BONGAERTS, J.H.H., FOURTOUNI, K. and STOKES, J.R. Soft-tribology: Lubrication in a compliant PDMS–PDMS contact. *Tribology International* [online]. October 2007. Vol. 40, no. 10–12, p. 1531–1542. DOI: 10.1016/j.triboint.2007.01.007.
- [112] LUGT, P. M. and MORALES-ESPEJEL, G. E. A Review of Elasto-Hydrodynamic Lubrication Theory. *Tribology Transactions* [online]. 31 March 2011. Vol. 54, no. 3, p. 470–496. DOI: 10.1080/10402004.2010.551804.
- [113] DE VICENTE, J., STOKES, J.R. and SPIKES, H.A. The Frictional Properties of Newtonian Fluids in Rolling–Sliding soft-EHL Contact. *Tribology Letters* [online]. December 2005. Vol. 20, no. 3–4, p. 273–286. DOI: 10.1007/s11249-005-9067-3.
- [114] MYANT, C., FOWELL, M., SPIKES, H. A. and STOKES, J. R. An investigation of lubricant film thickness in sliding compliant contacts. *Tribology Transactions*. 2010. Vol. 53, no. 5, p. 684–694. DOI: 10.1080/10402001003693109.
- [115] MYANT, C., SPIKES, H.A. and STOKES, J.R. Influence of load and elastic properties on the rolling and sliding friction of lubricated compliant contacts. *Tribology International* [online]. January 2010. Vol. 43, no. 1–2, p. 55–63. DOI: 10.1016/j.triboint.2009.04.034.
- [116] ASSAEL, M. J., BOTSIOS, S., GIALOU, K. and METAXA, I. N. Thermal Conductivity of Polymethyl Methacrylate (PMMA) and Borosilicate Crown Glass BK7. *International Journal of Thermophysics* [online]. September 2005. Vol. 26, no. 5, p. 1595–1605. DOI: 10.1007/s10765-005-8106-5.
- [117] GASNI, D., WAN IBRAHIM, M.K. and DWYER-JOYCE, R.S. Measurements of lubricant film thickness in the iso-viscous elastohydrodynamic regime. *Tribology International* [online]. July 2011. Vol. 44, no. 7–8, p. 933–944. DOI: 10.1016/j.triboint.2011.03.010.
- [118] MYANT, C., FOWELL, M. and CANN, P. The effect of transient motion on Isoviscous-EHL films in compliant, point, contacts. *Tribology International* [online]. April 2014. Vol. 72, p. 98–107. DOI: 10.1016/j.triboint.2013.11.019.
- [119] FOWELL, M.T., MYANT, C., SPIKES, H.A. and KADIRIC, A. A study of lubricant film thickness in compliant contacts of elastomeric seal materials using a laser induced fluorescence technique. *Tribology International* [online]. December 2014. Vol. 80, p. 76–89. DOI: 10.1016/j.triboint.2014.05.028.
- [120] SONG, K, WANG, Y, HUANG, B and WANG, M. Study on EHL of plastic pinion engaging with steel gear. 2005. P. 50- 51+54.
- [121] MARX, Nigel, GUEGAN, Johan and SPIKES, Hugh A. Elastohydrodynamic film thickness of soft EHL contacts using optical interferometry. *Tribology International* [online]. July 2016. Vol. 99, p. 267–277. DOI: 10.1016/j.triboint.2016.03.020.

- [122] KOCHI, T., ICHIMURA, R., YOSHIHARA, M., DONG, D. and KIMURA, Y. Film Thickness and Traction in Soft EHL with Grease. *JOURNAL OF JAPANESE SOCIETY OF TRIBOLOGISTS*. 2016. Vol. advpub. DOI: 10.18914/tribologist.16-00017.
- [123] SMEETH, M and SPIKES, H.A. The Influence of Slide/Roll Ratio on the Film Thickness of an EHD Contact Operating Within the Mixed Lubrication Regime. In : DOWSON, D, TAYLOR, C M, CHILDS, T H C, DALMAZ, G, BERTHIER, Y, FLAMAND, L, GEORGES, J.-M. and LUBRECHT, A A B T - Tribology Series (eds.), *The Third Body Concept Interpretation of Tribological Phenomena* [online]. Elsevier, 1996. p. 695–703. ISBN 0167-8922.
- [124] PUTIGNANO, C., REDDYHOFF, T., CARBONE, G. and DINI, D. Experimental investigation of viscoelastic rolling contacts: A comparison with theory. *Tribology Letters*. 2013. Vol. 51, no. 1, p. 105–113. DOI: 10.1007/s11249-013-0151-9.
- [125] PUTIGNANO, C., CARBONE, G. and DINI, D. Theory of reciprocating contact for viscoelastic solids. *Physical Review E*. 2016. Vol. 93, no. 4. DOI: 10.1103/PhysRevE.93.043003.
- [126] PUTIGNANO, C. and DINI, D. Soft Matter Lubrication: Does Solid Viscoelasticity Matter? *ACS Applied Materials and Interfaces*. 2017. Vol. 9, no. 48, p. 42287–42295. DOI: 10.1021/acsami.7b09381.
- [127] PANDEY, A., KARPITSCHKA, St., VENNER, C. H. and SNOEIJER, J. H. Lubrication of soft viscoelastic solids. *Journal of Fluid Mechanics* [online]. 2016. Vol. 799, p. 433–447. DOI: 10.1017/jfm.2016.375.
- [128] ZHAO, Yan, MORALES-ESPEJEL, G. E. and VENNER, C. H. Aspects of modeling and numerical simulation of dry point contacts between viscoelastic solids. *Tribology International* [online]. 2022. Vol. 165, p. 107245. DOI 10.1016/j.triboint.2021.107245.
- [129] ZHAO, Y., LIU, H. C., MORALES-ESPEJEL, G. E. and VENNER, C. H. Effects of solid viscoelasticity on elastohydrodynamic lubrication of point contacts. *Tribology International* [online]. 2022. Vol. 171, no. April, p. 107562. DOI: 10.1016/j.triboint.2022.107562.
- [130] OECD (2015), *Frascati Manual 2015: Guidelines for Collecting and Reporting Data on Research and Experimental Development* [online]. Paris : OECD, 2015. The Measurement of Scientific, Technological and Innovation Activities. ISBN 9789264238800.
- [131] KRUPKA, J., DOCKAL, K., KRUPKA, I. and HARTL, M. Elastohydrodynamic Lubrication of Compliant Circular Contacts near Glass-Transition Temperature. *Lubricants* [online]. 13 July 2022. Vol. 10, no. 7, p. 155. [Accessed 28 July 2022]. DOI: 10.3390/lubricants10070155.
- [132] KRUPKA, J., DOCKAL, K., KRUPKA, I. and HARTL, M. Polymer Lubrication: Pressure–Viscosity–Temperature Dependence of Film Thickness for Highly Loaded Compliant Contacts in Elastohydrodynamic Lubrication Regime. *Journal of Tribology* [online]. 1 February 2023. Vol. 145, no. 2. DOI: 10.1115/1.4055558.
- [133] KRUPKA, J., DOCKAL, K., SEDLACEK, T., REBENDA, D., KRUPKA, I. and HARTL, M. Viscoelastic Response of Elastohydrodynamically Lubricated Compliant Contacts below Glass-Transition Temperature. *Polymers* [online]. 30 May 2023. Vol. 15, no. 11, p. 2528. DOI: 10.3390/polym15112528.

- [134] KŘUPKA, J., ŠPERKA, P., KŘUPKA, I. and HARTL, M. LUBRICATION OF ROLLING - SLIDING POLYMER CONTACTS. In : *Engineering Mechanics 2019* [online]. 13 May 2019. p. 219–222.
- [135] CZECH SCIENCE FOUNDATION (GACR). Technical Sciences – Panel Specialization. [online]. 2022. [Accessed 2 August 2022]. Available from: <https://gacr.cz/en/panels/>
- [136] NUDEC S.A. PMMA SHEETS. [online]. 2023. Available from: <https://nudec-plastic.com/plastic-panels/polymethylmethacrylate-pmma-plastic-sheets/>
- [137] RGPBALLS S.R.L. RGPBALLS. [online]. 2023. Available from: <https://www.rgpballs.com/en/products/precision-balls/plastic-balls/>
- [138] INC., REDHILL PRECISION BALLS. Plastic Balls. [online]. 2023. Available from: <https://www.redhillballs.com/en-us/product/plastic-balls/>

NOMENCLATURE

Letters:

Symbols	Explanation and Units
A	Area of contact, mm ²
a _H	Hertzian contact radius, μm
a _T	Horizontal shift factor, -
b	Hertzian semi-contact width, μm
b _T	Vertical shift factor, -
C ₁ , C ₂	WLF constants
CoF	Coefficient of friction, -
D	Deborah number, -
E	Elastic modulus, MPa
E*	Complex modulus, MPa
E'	Storage modulus, MPa
E''	Loss modulus, MPa
E _A	Activation energy, J
E _R	Reduced elastic modulus, MPa
g _E (g ₃)	Parameter of elasticity, -
g _V (g ₁)	Parameter of viscosity, -
g ₄	Operational parameter, -
$\bar{G}$	Dimensionless parameter of material, -
H	Dimensionless film thickness, -
$\hat{H}$	Parameter of dimensionless film thickness, -
H _c	Dimensionless central film thickness, -
H _m	Dimensionless minimum film thickness, -
h _c	Central film thickness, nm
h _m	Minimum film thickness, nm
k	Ellipticity of contact, -
P	Parameter of pressure, (-)
P/S	Elasticity parameter, -
p	Contact pressure, MPa
p _{iv}	Asymptotic Isoviscous pressure, MPa
p _H	Maximum Hertzian pressure, MPa
q	McEwen coefficient, -
R'	Reduced curvature radius, mm
R	Curvature radius of specimens, mm
	Universal gas constant, J mol ⁻¹ K ⁻¹
R _c	Radius of static and running contact, μm
R _e	Yield strength, MPa
R _q	Root means square (RMS) roughness, μm
S	Speed parameter, -
T	Inlet temperature, °C
T _g	Glass-transition temperature, °C
T _m	Melting temperature, °C
T _s	Service temperature, °C

T_R	Reference temperature, °C
t	Time, s
$\tan \delta$	Loss factor, -
U_E	Entrainment speed, $\text{m}\cdot\text{s}^{-1}$
U	Surface speed of specimens, $\text{m}\cdot\text{s}^{-1}$
$\bar{U}$	Dimensionless parameter of speed, -
V_d	Abbe's number, -
W	Normal load, N
$\bar{W}$	Dimensionless parameter of load, -
x	Perpendicular direction to entrainment speed
y	Parallel direction to entrainment speed

Abbreviations:

Symbols	Explanation and Units
CoF	Coefficient of friction, -
SRR	Sliding-rolling ratio, -

Greek letters:

Symbols	Explanation and Units
α	Pressure-viscosity coefficient, GPa^{-1}
α_0	Initial pressure-viscosity coefficient, GPa^{-1}
α^*	Reciprocal Isoviscous pressure-viscosity coefficient, GPa^{-1}
α_{film}	Pressure-viscosity coefficient (suggested by Bair), GPa^{-1}
α_G	Pressure angle of the gear, deg
β	Thermal expansion, $10^{-6}\cdot^\circ\text{C}$
δ	Phase angle between stress and strain, rad
η	Dynamic viscosity of lubricant, Pa s
η_0	Dynamic viscosity of lubricant at atmospheric pressure, Pa s
Λ	Lubrication parameter, -
λ	Thermal conductivity, $\text{W}/\text{m}\cdot^\circ\text{C}$
ρ	Density of lubricant, kg/m^3
τ	Relaxation time of polymer, s
φ	Light transmittance, %
ν	Poisson's ratio, -

Subscripts:

Symbols	Explanation
1, 2	Values for PMMA disc and steel ball
alfa	Primary relaxation of the polymer
beta, gamma, delta	Secondary relaxations of the polymer
exp	Experimental values of
mod	Calculated values
mr	Values at the exit of contact
ms	Values at the side lobes of contact

LIST OF FIGURES

Fig. 1. Temperature dependence of elastic modulus E for the amorphous and semi-crystalline polymers.	15
Fig. 2. Stribeck's curve and lubrication regimes.	17
Fig. 3. Hydrodynamic map of elastohydrodynamic lubrication modes.	19
Fig. 4. Individual regions in the EHL contact.	19
Fig. 5. Hydrodynamic map of EHL for circular contacts operating in I-E mode. [73].....	23
Fig. 6. Hydrodynamic map for circular contacts covering I-E and I-R modes. a) Johnson [73], b) Hooke [100].....	26
Fig. 7. Contour plot of film thickness for circular and elliptical contacts and different P/S ratios. ($k = 2, 1$ and 0.5 , and $P/S=0.78;1.14$ and 1.67). Lubricant inlet from the left (see arrow). [100].....	27
Fig. 8. Dimensionless film thickness vs. distance across the contact for highly loaded compliant circular contacts. [100]	27
Fig. 9. Hydrodynamic map for different contact geometries. A) circular contact ($k = 1$), b) wide elliptical contact ($k = 6$). [96]	29
Fig. 10. Hydrodynamic map for the line contacts (according to Myers model). [95].....	30
Fig. 11. Contact pressure profiles for viscoelastic contacts. a) dry conditions, b) lubricated conditions. [103].....	31
Fig. 12. Film thickness profiles for different Deborah's numbers. ($g_E = 30$) Lubricant inlet from left. [103]	31
Fig. 13. Distribution of film thickness for different $U_E\eta$ ratios and loads W . Lubricant inlet from left. [105]	32
Fig. 14. Film thickness and pressure profiles. GLY lubricant. ($W = 3$ and 50 mN; $U_E = 0.65$ and 0.61 mm/s). [114]	33
Fig. 15. Comparison of the experimental central film thickness with the Hamrock & Dowson model. GLY lubricant. ($W=3, 35, 50$ mN; $U_E=0-1,8$ mm/s). [114]	33
Fig. 16. Film thickness and pressure profiles. Lubricant SFO and GLY. $\eta_{SFO} < \eta_{GLY}$. ($W = 3$ mN; $U_E = 0.66$ mm/s). [114]	34
Fig. 17. Distribution of the film thickness in compliant contact. Lubricant GLY. ($W = 3$ mN; $U_E = 0,66$ mm/s). [114]	34
Fig. 18. Film thickness profiles vs distance as function of entrainment speed. ($W = 3.8$ N, $p \approx 1.04$ MPa). Lubricant inlet from the left. [117].....	35

Fig. 19. Film thickness vs. entrainment speed as function of load. [117].....	35
Fig. 20. Film thickness profiles: static contact (red), stabilized lubricating film (dashed red), thickness course in the center of contact (black). Lubricant inlet from left. a) $1000 \text{ mm}\cdot\text{s}^{-2}$, b) $5 \text{ mm}\cdot\text{s}^{-2}$. [118].....	36
Fig. 21. Film thickness profiles at different sliding speeds: a) $75 \text{ mm}\cdot\text{s}^{-1}$, b) $10 \text{ mm}\cdot\text{s}^{-1}$. Lubricant inlet from the left. after $t = 5 \text{ s}$ (red), stabilized film thickness before sudden stop (dashed red), film thickness profiles at individual times t (black). [118]	36
Fig. 22. Comparison of film thickness of different gearsets (PEEK/STEEL and PEEK/PEEK). [10].....	37
Fig. 23. Comparison of film thickness depending on load for combination (PEEK/PEEK). [10]	37
Fig. 24. CoF vs. angular velocity for different gearsets. a) PEEK/PEEK, b) PEEK/steel. [10]	38
Fig. 25. Effect of entrainment speed on film thickness distribution in the compliant contact. Lubricant: GTL8; $W = 10 \text{ N}$; $\text{SRR} = 0$. [121].....	38
Fig. 26. The central film thickness h_c vs. different SRR ratios ($W=10 \text{ N}$). [121].....	39
Fig. 27. The effect of the load on the central film thickness. ($\text{SRR} = 0$). [121].....	39
Fig. 28. Minimum film thickness vs. entrainment speed. (steel/PMMA; $\text{SRR} = 0$, $W = 5 \text{ N}$). [121]	40
Fig. 29. Comparison of experimental data of central film thickness with soft EHL models. ($\text{SRR} = 0$). [121].....	40
Fig. 30. Experimental equipment used. a) rotary optical tribometer (Brno University of Technology, Czech Republic), b) Bruker 3D optical profilometer (Bruker, USA), c) HAAKE RotoVisco1® (Thermo Fisher Scientific, USA), d) HPV (Brno University of Technology, Czech Republic), e) Hysitron TI Premier Nanoindenter (Bruker, USA), f) DMA1 (Mettler Toledo, Switzerland).	53
Fig. 31. Selected semicrystalline polymer balls. From the left: PA 66, PEEK and POM. .	55
Fig. 32. The selected reference lubricants used in experiments.	57
Fig. 33. Simulation of the compliant contact. a) soft disc / hard ball, b) hard disc / soft ball.	59
Fig. 34. Comparison of the initial topography of the steel and polymer ball specimens. a) 100Cr6, b) POM.....	60
Fig. 35. The determination of the refractive index of lubricant. a) Abbe refractometer (ABBE-REF 1), b) interface of the air-lubricant, c) determination of the n using Abbe refractometer	61

Fig. 36. Rotary optical tribometer in the ball-on-disc configuration.....	62
Fig. 37. Optical chromatic interferometry. a) sensing process, b) principle and calibration	63

LIST OF TABLES

Table 1. Material properties of selected polymers and 100Cr6.....	13
Table 2. Prediction formulae of dimensionless central film thickness in I-E and P-E modes of EHL for elliptical/circular contacts. a) [97], b) [91], c) [93], [100], d) [114], e) [121]..	46
Table 3. Prediction formulae of dimensionless minimum film thickness at the contact exit hmr and on side lobes hms in I-E and P-E modes of EHL for elliptical/circular contacts a) hmr [97], b) hmr [91], c) hmr [93], [100], d) hms [93],[100], e) hmr [114], f) hmr [121].	46
Table 4. Comparison of material properties of different amorphous materials at ambient room temperature.....	54
Table 5. Material properties of selected semi-crystalline polymer balls.	56
Table 6. Precision grades of selected semi-crystalline polymer balls.	56
Table 7. Selected properties of reference lubricants (40 °C, p = 0.1 MPa).....	58
Table 8. Surface roughness analysis of ball specimens - initial state (before polishing)... ..	59
Table 9. Comparison of the topography of PA 66 balls before and after polishing process.	60
Table 10. Comparison of the topography of PEEK balls before and after polishing process.	61
Table 11. General overview of the experimental conditions of the research conducted....	65